# MONTHLY NOTICES

OF THE

# ROYAL ASTRONOMICAL SOCIETY.

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J. R. HIND, Esq., President, in the Chair.

Thomas Buckney Esq., 61 Strand, W.C. ;

Lindsay Atkins Eddie, Esq., Oatlands, Grahamstown, Cape of Good Hope ;

Thomas Gullon, Esq., Northenden, Cheshire ; and

Signor N. Perini, 261, Hampstead Road, N.W. ;

were balloted for and duly elected Fellows of the Society.

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*Notice.*—The Council have made arrangements for distributing by post to Fellows of the Society Ephemerides of Comets and other information of immediate interest. Any Fellow who may wish to receive such information is requested to forward his name to the Assistant Secretary, Royal Astronomical Society, Burlington House, London, W.

It is desired that early information respecting Comets may be sent direct to W. H. M. Christie, Royal Observatory, Greenwich, London, S.E., in order that it may be distributed without delay.

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*On the Preparations to be made for observation of the Transit of Venus* 1882, *December* 6. By Sir G. B. Airy, K.C.B., Astronomer Royal.

Whatever we may consider as concluded from the observations of the Transit of *Venus*, 1874, I think we may well believe that the scientific world will not be satisfied unless we take the opportunity of securing all that can be obtained from the Transit

of 1882: such an opportunity as will recur only after the extinction of three generations of mankind. It is not now too early to examine into the circumstances of that Transit, and to decide provisionally on the sites to be adopted, the observations that may most advantageously be taken, and the means of carrying them out.

We approach this Transit under conditions far more favourable than those under which we undertook the observations of 1874. We have the personal experience of observers who are still living among us, and our minds are still occupied with the discussions to which their observations gave rise. In external circumstances a great step has been gained by the extension of electric telegraphs, upon which we may rely for delivering us from that which was the most laborious and most expensive part of the Transit of 1874—namely, the determination of longitudes of stations. And, finally, I believe I can show that, by departing in some respects from the plan which I proposed many years ago, and laying greater stress upon considerations which I then treated more lightly, we may adopt a course which seems to possess all the elements of success.

I wish first to call attention to this point: that a most important condition for trustworthy observation is a sufficient elevation of the Sun above the horizon. For this advantage it will be well sometimes to sacrifice something in the magnitude of the parallax-factor; especially where observations can be obtained at several neighbouring stations: I am inclined to fix upon the elevation 20° as one that ought to be secured if possible: though an elevation of 15° may sometimes be accepted as sufficient.

This being premised, I proceed to indicate the stations which I consider most advantageous.

## *Transit of Venus* 1882.

### Ingress Accelerated.

For this phenomenon, I propose to rely entirely on the Cape Colony, employing as many stations as can be manned from the Cape Observatory to D'urban.

As regards absolute longitude: relying upon that of Aden (determined in 1874), and remarking that telegraph is complete thence to D'urban, and that all authorities—and especially the Telegraph Company—are anxious to promote this determination, I cannot doubt that, under Mr. Gill, the longitude of the Cape Observatory will be quickly ascertained. Then as regards the actual observation of the Transit: telegraph exists along the whole coast, and all observations will be at once referred to Cape Observatory time, and thus to Greenwich time.

The Sun's elevation ranges from about 33° to about 46°. (At

Kerguelen and Mauritius the elevation is only 10° : at the south point of Madagascar it is 21°.)

The factor of parallax is about 0·65. This is the smallest that I propose to use ; but its smallness is, in my opinion, abundantly compensated by the other very favourable circumstances.

The Ingress will also be visible in Britain, Belgium, France, Spain, and part of Africa. At Land's End and Valentia the Sun's elevation will be about 14°, at Greenwich about 10°, with factor 0·65. For the other countries, the circumstances are less favourable.

### Ingress Retarded.

I proposed formerly to refer principally to the coasts of the Canadian Dominion and the United States of North America. But (without wishing to discourage observations there) I now think that the elevation of the Sun, 15° to 18°, is too small. I propose to substitute the whole chain of the West India Islands, from the eastern extremity of Cuba to Barbadoes : or stations on the neighbouring continent of Central America. The Hydrographic Office of the United States has determined with great accuracy the longitude of Havana, Santiago de Cuba, Kingston (Jamaica), Aspinwall, Panama, San Juan de Puerto Rico, St. Thomas, St. Croix, St. John of Antigua, St. Pierre of Martinique, Bridgetown of Barbadoes, Port Spain of Trinidad. At any of these it will be only necessary to determine local time at the observation.

The Sun's elevation ranges from about 23° to 43°.

The factor of parallax from about 0·75 to 0·85.

Bermuda also is very favourably situated, with Sun's elevation about 25° and factor of parallax about 0·90. But I believe that its longitude is not so well determined.

### Egress Accelerated.

All the stations suggested for Ingress retarded may be adopted with advantage for Egress accelerated. The Sun's elevation at Bermuda will be about 13°, that for Charleston 22°, that for the chain of islands about 25°, and that for the continental stations about 35°. The parallax-factor ranges from about 0·7 to 0·9.

### Egress Retarded.

Although the Egress can be observed at Melbourne with elevation 10°, and at Sydney and the whole eastern coast of New South Wales with elevation 14°, and the French will probably adopt New Caledonia with 22° elevation, yet I should be inclined to rely more completely upon New Zealand. There is nothing to prevent determination of the longitude of points of Australia, if a proper department of State would take it up, the telegraphic

communication being complete. For that of New Zealand, though considerable progress was made by Major Palmer in a series of Lunar Transits at Burnham, yet I think them scarcely sufficient for our purpose; and, if no telegraph be completed, I would propose some runs of chronometers, on which I have learned to place much reliance.

The Sun's elevation on the east coast of Australia (as stated above) would be perhaps 14°; that at Hobart Town 14°; that in New Zealand 30° to 35°.

The factor of parallax in Australia about 0·96; that in New Zealand about 0·83.

I will now advert to the instruments to be carried out.

For Local Time, at a station where the residence is to be one of considerable length, no instrument is comparable to the ordinary transit-instrument. But for short residences, intended to be no longer than is necessary for determining the state of the clock, and (if required) the local latitude, the most convenient instrument is a vertical circle, or altazimuth with no azimuth-circle or no accurately-divided azimuth-circle. (If, however, a survey for defining the place for observation should be contemplated, the azimuth-circle would be useful.) By using the vertical circle on two known stars in azimuth distant nearly 90° (it matters not in what portions of the azimuthal circumference nor at what elevation), latitude and clock-error are determined without difficulty. I should recommend the adoption of this instrument, where it can be obtained, in preference to any other. As everything depends upon its fitness for use, it ought to be furnished with duplicates of the parts most likely to be injured or lost—as levels and eye-pieces.

A good sidereal clock is indispensable, and a solar chronometer almost equally necessary.

For observation of the Transit, there ought to be a 6-inch telescope equatoreally mounted, with clock-work movement, and with careful provision in the eye-piece against the sun's radiant heat. Experiments on the efficiency of this protection ought previously to be made in the heat of the summer sun. And, if possible, another 4 or 5-inch telescope, similarly protected, ought to be furnished. A portion of these instruments can at once be supplied from the Royal Observatory.

It is decided, by almost unanimous consent, that no real assistance can be obtained by application of photography.

I should endeavour to dispense with framed huts and revolving roofs, thinking it possible that we may sufficiently trust to waterproof cloth coverings, or to such buildings (greenhouse, for instance, or ordinary shed) as may be found on the spot.

If at any station another competent observer can be found, and if the climate is uncertain, and if communications of time can be kept up (but not otherwise), a separation of the telescopes might

be advantageous. And it may be an important element of success that there be easy communication with the telegraph-office.

I have only further to remark that it is desirable that the history of the phenomena noted at each of the stations of past Transits should be carefully perused in making arrangements for this which is approaching. And I avow that some injury resulted to the observations in 1874 for want of this preparation. Attention had been so completely concentrated on the Transit of 1769, as affording a determination of Solar Parallax free in great measure from the effect of doubts on longitude, that the observations made in the Transit of 1761 were almost entirely neglected. But the Transit of 1761, partly visible in Europe, was, in fact, more extensively observed, and perhaps by abler professional observers, than that of 1769. There are numerous records of the observations of 1761 in the following volumes of the *Philosphical Transactions*. Among the results there is one—namely, the ring of light round the planet *Venus*—which, had it been known to our observers, would have saved them from some trouble, and possibly some inaccuracy, in the observations of 1874.

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*Note in Correction of a Passage in his Paper "On the Theoretical Value of the Acceleration of the Moon's Mean Motion in Longitude" (Monthly Notice* 1880, *April)*. By Sir G. B. Airy, K.C.B, Astronomer Royal.

It has been pointed out to me by Professor Adams that the second part of Paragraph 16 in the paper on Acceleration has no foundation. This is perfectly clear, inasmuch as the factor $\frac{\sigma}{A^3}$, which is proportional to $\frac{1}{T^2}$ (T being put for the length of the year), is invariable so long as the year is invariable. With change of A, $\sigma$ must be changed. I cannot explain the occurrence of this error, except by the remark that the paragraph in question was written last of all—a circumstance which, as I well know, tends powerfully to increase probability of error.

1880, *May* 15.

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*On the Determination of the Solar Parallax by Means of the Parallactic Inequality in the Motion of the Moon.* By James Campbell, Esq., and E. Neison, Esq.

As the accurate determination of the true value of the distance of the Sun is one of the most pressing requirements of modern astronomy, and as the various attempts to determine its value which have been made of late years have only resulted in increasing the uncertainty as to its real value, it seems to be most important to carefully discuss each method which can be used for determining the true value of the solar parallax, with the view of ascertaining the most advantageous manner of employing the method, and of determining, in the instances where it has already been employed, whether any systematic error has vitiated the results which have been obtained. In the present paper is presented such a discussion of the method of determining the solar parallax by means of the parallactic inequality in the motion of the Moon.

Viewed from its theoretical aspect, the most feasible method which can be employed at the present time for accurately determining the distance of the Sun is undoubtedly afforded by the parallactic inequality in the motion of the Moon. For in this inequality the solar parallax is multiplied by a coefficient which increases the quantity to be determined by nearly fifteen, so that by deducing from observation the value of the parallactic inequality to only one-seventh of a second of arc, it would enable the solar parallax to be ascertained to within less than one-hundredth of a second of arc. If, therefore, it were possible to deduce from observation the true value of the parallactic inequality, it would furnish the most advantageous method of ascertaining the true distance of the Sun that is likely to be at our disposal for many years. It is true that in time it will have to yield this theoretical advantage to the methods founded on the determination of the amount of the secular variations in the elements of the orbits of the planets, but it will be probably a century before this is the case.

When the first of these methods comes to be applied in practice, a very grave difficulty is encountered—so grave a difficulty that it has been regarded as insurmountable by modern investigators, and held to be fatal to this method of determining the solar parallax. This difficulty is due to the fact that when we discuss a series of observations of transit of the Moon's limb, what is obtained is the correction, not to the true value of the parallactic inequality, but the difference between the correction to the true value of the parallactic inequality and a small periodical variation in the apparent semi-diameter of the Moon due to the varying contrast between the brightness of the lunar limb and that of the sky which it is seen against. When the parallactic inequality tends to increase the Moon's longitude, then

this variation in the apparent semi-diameter tends to decrease the longitude; and when this variation in semi-diameter tends to increase the longitude, then the parallactic inequality tends to decrease it. Accordingly, the *apparent* value of the parallactic inequality as immediately derived from the observations is systematically less than the *true* value, which is what is required for determining the distance of the Sun.

To obtain the *true* value of the parallactic inequality from the *apparent* value furnished by the discussion of transits of the Moon's limb, it is necessary to apply a correction for the effect of this variation in the apparent semi-diameter of the Moon. The whole difficulty lies in deducing the proper value of this correction, and it has hitherto been found to be impracticable to do this. It may amount to anything between two tenths and twenty tenths of a second of arc, and to the moiety of this extent the parallactic inequality must be regarded as uncertain.

A number of attempts have been made to utilise this method of determining the solar parallax, by endeavouring to deduce the true value of the parallactic inequality from a discussion of the observations of the Moon, and then by comparing this with the theoretical value of the inequality to ascertain by how much the theoretical value of the solar parallax requires to be altered in order to give the real value. Rejecting some of the very early attempts, the principal determinations which have been made of the value of the parallactic inequality appear to be the following:—

1812 Burckhardt. From a discussion of over 4000 early Greenwich observations, the value 123″·5 was obtained. But the value of the lunar semi-diameter employed by Burckhardt was fully 2″·0 too small, and the effect of this error would be to decrease the apparent value of the parallactic inequality by nearly 1″·0, so that its apparent value from Burckhardt's discussion would seem to be really 124″·5.

1848 Airy. From a discussion of the Greenwich Lunar Observations made between 1750 and 1830, the value 122″·37 was deduced for the value of the coefficient of this inequality. This was regarded as very uncertain.

1854 Hansen. From a discussion of a number of observations made at Greenwich and at Dorpat, the value 126″·46 was obtained for this coefficient.

1859 Airy. From a discussion of the entire series of meridian observations of the Moon made at Greenwich between 1750 and 1851 there was deduced a value of 124″·79. From four years' observations with the Greenwich altazimuth instrument, the value 125″·50 was deduced. As considerable uncertainty existed with regard to the proper value of the Moon's semi-diameter to be used in the early observations, the Astronomer Royal considered it better to use only the meridian observations made since the year 1811. These gave the value 124″·37. Finally, Sir G. Airy expressed his conviction that the value of the parallactic inequality could not differ much from 124″·70.

1867 Stone. From the discussion of over 2000 observations made at Greenwich between 1847 and 1866, the value 125″·36 was obtained.

1868 Newcomb. From a discussion of the Washington observations made during the four years 1862–1865, the value 125″·46 was obtained.

When these are examined it will be found that the apparent discordance between these values is to a certain extent fictitious, and arises from some being the values of the apparent coefficient of the parallactic inequality, and others the values of the real coefficient of the inequality. Properly separating these, they are

| | Apparent Coefficients. | Real Coefficients. |
|---|---|---|
| | ″ | ″ |
| Burckhardt | 124·5 | |
| Airy | 122·37 | |
| Hansen | | 126·46 |
| Airy | 124·70 | |
| Stone | | 125·36? |
| Newcomb | 124·36 | 125·46 |

The four last values only need retain our attention, and these we proceed to examine. For convenience, let the following notation be employed:—

$l$ = mean longitude of the Moon,

$l'$ = mean longitude of the Sun,

$\alpha$ = mean anomaly of the Moon,

$\mu$ = mean anomaly of the Sun,

$\beta$ = mean departure of the Moon from the ascending node of its orbit,

$\theta$ = the difference between the mean longitude of the Moon and Sun.

Further for brevity put

$$\begin{aligned} \mathrm{P}\sin p = \mathrm{P}\sin\theta &= \text{Parallactic Inequality,} \\ \mathrm{V}\sin v = \mathrm{V}\sin 2\theta &= \text{Variation,} \\ \mathrm{M}\sin\mu &= \text{Annual Equation.} \end{aligned}$$

Next let $\mathrm{P}_o$, $\mathrm{V}_o$, $\mathrm{M}_o$, denote the values of the coefficients P, V, M which are employed in the tables of the Moon with which the observations are compared, and let $\delta\mathrm{P}$, $\delta\mathrm{V}$, $\delta\mathrm{M}$, denote the corrections which these values require, these corrections being defined by the equation

$$\mathrm{P} + \delta\mathrm{P} = \mathrm{P}_o,$$

so that they indicate the augmentation of P, &c., necessary to bring it up to the value used in the tables.

Finally, suppose that (P), (V), (M) denote the apparent values of these coefficients—that is to say, the values which would be obtained from a discussion of the observations of the Moon without the elimination of the systematic errors due to variation in the apparent diameter of the Moon, &c., &c.

The first step to be taken is to ascertain what precautions ought to be used to ensure the accuracy of the value found for the coefficient of the parallactic inequality by a discussion of the observations. It will then be easy to see whether any of these precautions have been neglected in the determinations which have already been made of the value of the coefficient of this term.

It will be well to ascertain first of all the conditions which ought to be fulfilled by the lunar tables with which the observations are compared, as it is from the discordances between the tables and the observations that the corrections are to be deduced. The imperfections in the tables may be divided into the following:—

I. Imperfections in the theory of the terms of very long period and in the values of the secular mean motion and secular acceleration. In the discussion of the corrections to the terms of short period like the parallactic inequality, the imperfections of this class are of no importance, as they can be completely eliminated.

II. Imperfections in the theory of the terms of moderately long period. In general, errors in the values of these terms will not be of importance, the only exception being in the case of the annual equation, as this may have considerable indirect influence. For the annual equation has the effect of decreasing the longitude of the Moon in the spring of the year and of increasing its value in the autumn; whereas most observations of the Moon when the parallactic inequality is positive are obtained in the spring, and most when the parallactic inequality is negative are obtained in the autumn. The effect of this is to throw a portion of the correction to the annual equation on to the correction to the parallactic inequality. For suppose in the spring, when the argument of the annual equation is positive, that out of 50 observations 30 are made before Full moon, when the argument of the parallactic inequality is positive, and only 20 are made after Full moon, when the argument of the parallactic inequality is negative; then, the errors of the tables for these observations being $a'$ and $b'$, equations would result of the form

$$30\{\delta P\} + 30\{\delta M\} = a', \qquad 20\{-\delta P\} + 20\{\delta M\} = b',$$
$$\therefore \quad 50\{\delta P\} + 10\{+\delta M\} = a' - b'.$$

Similarly, if in the autumn, when the argument of the annual equation is negative, the numbers be supposed to be reversed, then the equations will be

$$20\{\delta P\} + 20\{-\delta M\} = a'', \qquad 30\{-\delta P\} + 30\{-\delta M\} = b'',$$
$$\therefore \quad 50\{\delta P\} + 10\{\delta M\} = a'' - b''.$$

Consequently, to determine $\delta P$, there exists the equation

$$100\{\delta P\} + 20\{\delta M\} = (a' - b') + (a'' - b'').$$

It follows, therefore, that in the above case one-fifth of the amount of any error in the annual equation would be thrown on the value found for the coefficient of the parallactic inequality. For this reason it is necessary that the tabular value of the apparent annual equation should be accurate, or its error be determined. In forming the above equation it has been assumed that the coefficients of the factors $\delta P$, $\delta M$ are unity for each observation: this is not the case, so that the values 30 and 20 ought to be replaced by the sum of this number of coefficients of these quantities.

III. Imperfections in the theory of the terms of short period which are not nearly coincident in period with the parallactic inequality, nor with a multiple of it. These terms constitute nearly ninety-nine hundredths of the terms in the lunar theory, and unless they are affected by errors of large magnitude, their imperfections are without influence on the accurate determination of the parallactic inequality. This is in consequence of their mutually destroying each other when a number of observations are united. In determining, therefore, the value of the parallactic inequality by comparison with any modern tables, it may be assumed that the tabular values of these inequalities are correct. In fact, in the case of Hansen's tables it is probable that the error in these coefficients is rarely sensible.

IV. Imperfections in the theory of the terms of short period which have periods nearly coincident with that of the parallactic inequality, or of a multiple of it. These terms require careful consideration.

Suppose that the theory of the motion of the Moon in longitude which forms the basis of the tables is imperfect owing to its not containing a small variable term with a period differing from that of the parallactic inequality by the small quantity $b$, this quantity $b$ being an angle increasing by a few degrees per annum. The tabular longitude of the Moon would thus be systematically incomplete by this term, which may be denoted by

$$B \sin (\theta + b).$$

As $\delta P$ denotes the correction required by the tabular coefficient of the parallactic inequality, when the tables are compared with the observations, they will be found to be discordant by the quantity

$$\delta P \sin \theta + B \sin (\theta + b) = \{\delta P + B \cos b\} \sin \theta + B \sin b \cos \theta, = E.$$

Suppose at the epoch of any observation the value of $\sin \theta$ be $= a$, then the value of the coefficient of $\sin \theta$, or of the supposed correction to the parallactic inequality, will be given by the equation

$$\delta[P] \times a = E.$$

In reality, what would be equated to E would be

$$\{\delta P + B \cos b\} \times a = E.$$

When the sum of a number of observations is taken, the term depending on cos $\theta$ will be entirely destroyed by the cases where cos $\theta$ is negative destroying those where it is positive, so that it may be neglected. But the quantity $\delta$ [P] which would be obtained as the correction to the parallactic inequality from a year's observations would not be the true correction to this term, but would be the sum of the true correction and the mean value for the year of the quantity B cos $b$. Suppose the existence of this term B sin $(\theta+b)$ to be unsuspected. Then each year would yield a different value to the correction to the parallactic inequality owing to the variation in the value of the factor B cos $b$ as the argument $b$ increases from year to year; and, on comparing these yearly values of the correction to the parallactic inequality, they would show a periodical variation with a period equal to that of $b$. Therefore, if $b$ had a moderately long period, so that the observations from which the correction to the coefficient of the parallactic inequality was to be deduced extended only over a fractional portion of this period, the resulting value would be erroneous by the mean value of the factor varying with $b$. If $b$ had merely a comparatively short period, so that the observations extended over several periods of $b$, then its effects might be considered as eliminated from the mean of all, as the positive values at one part of the period would be destroyed by the negative values at the other part. But even then the existence of this term would give rise to discordances in the results for separate years, which would have the effect of throwing doubt on the accuracy of the resulting mean value. It is essential, therefore, that the lunar tables should contain no such imperfection, or that means should be taken to discover the existence of any such error and to determine its effect on the results.

When the lunar theory is examined in order to see if there be any terms of this nature, it will be found that the perturbations due to the direct action of the Sun contain only three sensible inequalities which have periods nearly coincident with that of the parallactic inequality. Any error in the value assigned to the coefficients of these terms will produce, obviously, the same effect as the existence of a new term with the same argument and with a coefficient equal to the quantity which must be added to the tabular value in order to render it accurate. These three terms are those with the argument

$$\alpha-\mu, \qquad 2\beta-\alpha-\mu, \qquad 2\theta-\alpha+\mu.$$

The periods of the first and third of these differ from that of the parallactic inequality by the difference between the mean motion of the perigees of the Moon and Sun, a quantity which increases by over 40°·6 per annum. This quantity is so large that the observations of a few years would neutralise the effect of any small errors in these coefficients. As the theoretical value of these terms is easily computed, it is highly improbable that there is any sensible error in the modern values employed in

the lunar tables. It may be safely assumed, therefore, that the accuracy of the value of the parallactic inequality will not be affected by errors in the coefficients of these two terms. The second term has a period differing from that of the parallactic inequality by less than 2° per annum. It would affect the value found for the parallactic inequality by a most dangerous systematic error. It would be an error very difficult to detect, and it would have the effect of making the coefficient of the parallactic inequality appear too large for nearly a century, and too small for the next century. Fortunately, it is a term which has so small a coefficient—less than one-fifth of a second of arc—that it can be regarded as nearly insensible, whilst it is not difficult to determine its true value with great accuracy; so that it is improbable that its theoretical value is inaccurate by as much as one twenty-fifth of a second of arc. It may therefore be safely neglected.

The only important terms of the kind having arguments close accordant in period with that of some multiple of that of the parallactic inequality are those with the arguments

$$2\theta \qquad 3\theta \qquad 4\theta \qquad 5\theta \qquad 6\theta.$$

All these are exact multiples of the argument $\theta$ of the parallactic inequality. It may be stated that the terms with arguments differing by a small quantity from that of a multiple of the argument $\theta$ are far less important than those differing from $\theta$ itself by a small quantity. For they introduce a sensible error during a part only of the period, and that error more quickly vanishes. But a still more important reason is the fact that they are nearly all extremely small; and, as their values are very easily calculated, it is most improbable that the modern theoretical values are erroneous to any important amount.

Of the terms whose arguments are given above, those with the arguments $4\theta$ and $6\theta$ may be neglected, as not only is it unlikely that they involve any sensible error, as they are small, but the effect of any error is almost completely eliminated when a number of observations are combined.

The terms depending on the arguments $3\theta$ and $5\theta$ might exert very considerable effect on the value found for the parallactic inequality if their coefficient were sensibly erroneous. The term depending on $5\theta$ may be, however, disregarded at once, for its theoretical value is unquestionably less than two hundredths of a second of arc, and in all the tables it is regarded as insensible. The term with the argument $3\theta$ is more important, for it has a most systematic effect on the value of the parallactic inequality. Suppose the parallactic inequality is to be determined from observations made within two days on either side of the first or third Quarters of the Moon's age. When the Moon is within two days of the first Quarter, the value of $\theta$, the argument of the parallactic inequality, will range between 65°

and 115°, and sin $\theta$ will range between the values +0·90 and +1·00, its mean value being about +0·95. During the same period the value of $3\theta$ must be between 195° and 345°, so that sin $3\theta$ will range between −0·26 and −1·00, and its mean value may be taken at −0·71. Consequently, if $\delta P$ and $\delta Q$ denote the errors in the coefficients of these terms, they will produce an average error in the tabular place of the Moon at about the first Quarter amounting to

$$0{\cdot}95\,\delta P - 0{\cdot}71\,\delta Q.$$

Similarly, when the Moon is within a like period on either side of the third Quarter, the values of $\theta$ will range between 245° and 295°, and the mean value of sin $\theta$ will be about −0·95, whilst the values of $3\theta$ will range between 15° and 165°, and the mean value of sin $\theta$ be about +0·71. Therefore, the error produced will average at the third Quarter

$$-0{\cdot}95\,\delta P + 0{\cdot}71\,\delta Q.$$

The difference D between these two values will be, therefore,

$$D = +1{\cdot}90\,\delta P - 1{\cdot}42\,\delta Q;$$

whence

$$\delta P = +0{\cdot}53\,D + 0{\cdot}75\,\delta Q.$$

If $\delta(P)$, therefore, be the value of the correction found by neglecting the effect of any error in the coefficient of the term with the argument $3\theta$, the true value of the correction will be

$$\delta P = \delta(P) + \frac{3}{4}\delta Q.$$

When the parallactic inequality is determined from the entire mass of observations, but double weight is given to all observations in which the value of sin $\theta$ is large, it can be shown in exactly the same manner that a greater part of this error is eliminated and that the true correction is

$$\delta P = \delta(P) + \frac{1}{3}\delta Q.$$

As it is possible to determine the value of the coefficient Q of this term with great accuracy, the error in the tabular value of the coefficient can be easily discovered and the proper correction applied.

The only other term requiring consideration is the *Variation*, the argument of which is denoted by $2\theta$. If the observations from which is to be determined the value of the parallactic inequality were equally distributed on either side of the first and third Quarters, the effect of any error in the tabular value of this term would be zero; but unless a great number of observa-

tions are rejected this will not be the case, as far more observations are made after the first Quarter and before the third Quarter than before the first and after the third Quarter, so that in the sum of the observations made before and after each Quarter the effect of an error in the variation will not be completely eliminated. This outstanding error will be negative at the first Quarter and positive at the third Quarter, so that when the difference between the tabular errors at these times is taken, these two portions will coalesce, and a portion of the error in the coefficient of the variation be thrown on the correction to the coefficient of the parallactic inequality. The method of approximately determining the amount of the error thus introduced can be determined exactly as in the case of the term with the argument $3\theta$. If $\delta V$ be the error in the tabular value of the coefficient of the variation, the correct value of $\delta P$ will be given by one of the following two expressions.

If the correction $\delta(P)$ be deduced from the difference between the sum of the tabular errors within two days on either side of the Quarters,

$$\delta P = \delta(P) + \frac{1}{9}\delta V.$$

If the correction $\delta(P)$ be deduced from all the observations, weighted as stated before,

$$\delta P = \delta(P) + \frac{2}{3}\delta V.$$

It may here be stated that these values are perfectly in accordance with the actual coefficients found from a discussion of a large number of observations, as will be shown subsequently. For this reason, unless only observations are employed which are equally distributed on both sides of the Quarters it will be essential to correct the resulting value of the correction to the parallactic inequality for any error in the coefficient of the variation.

V. Imperfections in the tabular constants. For the present purpose the only constant in which an error would be important is the semi-diameter. If the tabular semi-diameter be too great by the quantity $\delta S$, it will have the effect of increasing the longitude before Full Moon, and decreasing the longitude after Full Moon. As this is exactly contrary to the effect of the parallactic inequality, it will tend to make the observed value of the parallactic inequality too small. For this reason the observed value (P) should receive a correction and the true value will be

$$P = (P) - \delta S.$$

The tabular value of the semi-diameter must represent the apparent value of the semi-diameter of the Full Moon, and should afford the exact interval between the transit of the two limbs when the Moon is Full. The variation in the semi-diameter due

to the varying contrast in the brightness of the Moon's limb and the sky is the most important factor in the accurate determination of the value of the coefficient of the parallactic inequality, and has considerable influence on the apparent values of other of the inequalities in the motion of the Moon. It is a correction, however, rather to be applied to the observations than to the tabular place of the Moon.

The practical difficulties which occur in the treatment of the investigation will be examined as they occur.

The second step to be taken is to examine the methods which have been used by previous investigators in order to see how far they have taken into account and eliminated any errors of the kind which have been discussed in the preceding.

In his investigations Sir G. Airy has employed Tables of the Moon partially constructed by himself and founded on Plana's development of the theory of the lunar perturbations. The tables are affected, therefore, by nearly all the errors contained in Plana's lunar theory. On comparing the theory of the Moon embodied in these tables used by Sir G. Airy with the more correct theory as developed in the investigations of Hansen and Delaunay, it will be found that the tables are affected by no less than three errors of nearly $4''$, by two errors of nearly $2\frac{1}{2}''$, and by two errors of $1\frac{1}{2}''$ (see Delaunay, *App. Conn. des Temps*, 1869), besides great numbers of smaller errors. In consequence, when these tables are compared with observations, they must exhibit large and rapidly varying discordances. Fortunately, none of these larger errors are in the coefficients of the terms which can systematically affect the value of the parallactic inequality.

In their several investigations Professors Hansen and Newcomb have both taken Hansen's *Tables de la Lune* as the basis of their work. These tables have shown a peculiar error in the value assigned to the mean longitude of the Moon, but in other respects accord far closer with observation than any other tables have done. The values found by Hansen for the coefficients of the terms of short period are with rare exceptions in close accord with the values found by Delaunay and by Pontecoulant when the theory of the latter has been extended far enough. In fact, there is reason to believe that these values represent with great accuracy the true perturbations of the Moon produced by the direct action of the Sun. A few of the terms may require very small corrections, and so may some of the coefficients which, like those of the parallactic inequality and equation of the centre, have been deduced from observation and not from pure theory. There may be also more than one term omitted from the tables.

Mr. Stone in his investigation has also nominally employed Hansen's tables as the basis of his labours, but for the greater part of the observations what has been employed is really a kind of Burckhardt-Hansen table. As the details of Mr. Stone's

investigation have never been published, there must remain some uncertainty as to the exact nature of the means employed. From 1862–1866 the Greenwich observations have been compared with Hansen's tables, and the observations employed by Mr. Stone are quite satisfactory. But it does not appear that the Greenwich Observations for the years 1859, 1860, and 1861, have ever been compared with Hansen's tables, and if any of these observations were used by Mr. Stone, it must have been as compared with Burckhardt's tables. For the period 1848–1858 Mr. Stone has probably used the comparisons between Hansen's tables and observations, given by the Astronomer Royal as an Appendix to the Greenwich Observations for 1859. But though the Greenwich Observations for these years are nominally compared with Hansen's tables, this is not strictly the case. The differences between observation and theory really consist of the difference between Burckhardt's tables and observation added to the difference between Burckhardt's tables and Hansen's tables *at midnight*. If, then, between midnight and the epoch of observation the difference between the tabular longitudes of Burckhardt and Hansen would have changed, the supposed comparison between Hansen's tables and observation would be as much in error. If, therefore, there are errors in Burckhardt's tables which change at the rate of $0''{\cdot}08$ per hour, as there well may be, then at the Quarters when the Moon is observed six hours away from midnight these errors would produce discordances amounting to nearly half a second of arc. As these errors would be systematic, it cannot be assumed that they are unimportant, and in deducing the value of the parallactic inequality it must not be assumed without careful examination that this indirect comparison may be regarded as equivalent to a direct comparison between Hansen's tables and observation.

It remains now to compare the differences between these tables in the case of the six terms which have been distinguished from the rest by their systematic effect on the parallactic inequality. These six terms are the following:—

| Argument | Values of the Coefficients according to Burckhardt | Airy | Hansen | Delaunay |
|---|---|---|---|---|
| $\mu$ | $-\ 673''{\cdot}3$ | $-\ 669''{\cdot}0$* | $-\ 669''{\cdot}83$ | $-\ 668''{\cdot}91$ |
| $\alpha-\mu$ | $+\ 147{\cdot}5$ | $+\ 148{\cdot}1$ | $+\ 148{\cdot}02$ | $+\ 148{\cdot}24$ |
| $2\beta-\alpha-\mu$ | $+\ 0{\cdot}2$ | $+\ 0{\cdot}0$ | $+\ 0{\cdot}08$ | $+\ 0{\cdot}12$ |
| $2\theta$ | $+2373{\cdot}4$ | $+2370{\cdot}7$* | $+2369{\cdot}86$ | $+2369{\cdot}74$ |
| $2\theta-\alpha+\mu$ | $-\ 27{\cdot}6$ | $-\ 28{\cdot}8$ | $-\ 28{\cdot}53$ | $-\ 28{\cdot}28$ |
| $3\theta$ | $+\ 2{\cdot}7$ | $+\ 0{\cdot}9$ | $+\ 0{\cdot}41$ | $+\ 0{\cdot}53$ |

The values marked with an asterisk under the heading Airy are not the tabular values, but these values *plus* the corrections deduced by Sir G. Airy from his discussion of the observations,

and are, therefore, tantamount to the values actually employed by him in determining the value of the parallactic inequality.

In comparing the values used by Burckhardt with those employed by Hansen, it must be remembered that, though there may be considerable discordances between the values employed in the two tables, we have to deal only with such discordances as will in the space of a few hours near the time of the Moon's Quarters cause sensible systematic differences in the tabular places of the Moon—that is to say, differences which will vitiate the assumption made by Sir G. Airy that there will be no sensible difference between the tables of Burckhardt and Hansen at midnight and an epoch six hours before or after midnight.

The values assigned to the coefficient of the three arguments $(a-\mu)$, $(2\beta-a-\mu)$ and $(2\theta-a+\mu)$ are so nearly alike that the effects of the differences may be considered insensible. In the case of the coefficient of the term with the argument $\mu$ (the annual equation) there is a difference of nearly $1''$ between the values employed by Airy and Hansen. The difference may exert a sensible influence on the values found for the parallactic inequality. Airy's value is really that of the apparent value of the annual equation as affected by any systematic error due to variation in semi-diameter, &c., and, being derived from the observations, was the proper value to be employed by him. The difference between Burckhardt and Hansen will not give rise to sensible variations in the space of a few hours, so that it is immaterial.

The value assigned by Airy to the coefficient of $2\theta$ differs by nearly $1''$ from the values found by Hansen and Delaunay; but here again Airy's value is that of the apparent variation, and Hansen's and Delaunay's that of the true variation. This difference must again give rise to systematic differences in the values found for the coefficient of the parallactic inequality. Airy's value being that given by the observations is the correct one to have been used by him. If the apparent variation has really a coefficient $0''\cdot9$ greater than the theoretical variation, then both Stone and Newcomb's values for the parallactic inequality require the correction

$$\frac{1}{9}\delta V = \frac{1}{9}\times\{-0''\cdot9\} = -0'\cdot10.$$

But in this case, as in that of the annual equation, it is the apparent coefficients which must be used in deducing the true values of the other coefficients—that is to say, the values as affected by changes in the apparent semi-diameter, &c., and as these variations will usually differ with different instruments, it is essential to determine the correction for the effect of any error in the variation and annual equation from the observations themselves. If there be introduced into the equations from which the value of the inequality is to be determined a correction for the effect of change in semi-diameter, then, of course, the true values of

the inequalities must be employed, and the results obtained will be the more accurate of the two.

Burckhardt's value of the variation differs by over $3''{\cdot}5$ from the value employed in Hansen's tables, and as this difference will change quickly, it is of importance. Its effects may be ascertained as follows:—

When the Moon is in its first Quarter it will transit about six hours before midnight, and as the argument of the variation increases at a rate of rather more than 1° per hour—so that on the average the argument of the variation will be 6° less at transit than at midnight—therefore, if $\delta V$ denote the excess of Burckhardt's value over that of Hansen, and as at first Quarter the argument of the variation has a mean value of 180°, the difference between the two tables at midnight will be greater than at six hours earlier by the quantity

$$\delta V \sin (180° + 6°) = +0{\cdot}105\,\delta V.$$

That is to say, that about the time of the first Quarter of the Moon's age, if B denotes Burckhardt's value of the longitude of the Moon at midnight, and H denotes Hansen's value at the same epoch, at the average time of the transit of the Moon, about six hours before midnight, the difference between the two tables would be

$$(H-B)-0{\cdot}105\,\delta V.$$

On the other hand, when the Moon is about the time of third Quarter, so that it would transit on the average about six hours after midnight, the argument of the variation at the time of transit would be on the average 6° greater than at midnight, and as the mean value of the argument of the variation at third Quarter is 180°, there would be at transit a similar difference of

$$\delta V \sin (180° + 6°) = -0{\cdot}105\,\delta V;$$

whence if $(H'-B')$ were to denote the difference between the two tables at midnight, at the time of transit, six hours later, the difference would be

$$(H'-B')+0{\cdot}105\,\delta V.$$

Now, the supposed difference between Hansen's tables and observation was found by adding the difference $(H-B)$ to the difference $(B-O)$, where O denotes the observed longitude, whereas the values given above ought to have been used. Consequently, instead of the residuals being really $(H-O)$ for the time of observation, they are really at the time of observation

$(H-O) + 0{\cdot}105\,\delta V$ at first Quarter, and

$(H'-O')-0{\cdot}105\,\delta V$ at third Quarter.

If, then, D be the difference between the errors of Hansen's

tables at the first and third Quarters, instead of obtaining this quantity by subtracting the two expressions given above, what will be obtained will be

$$D + 0{\cdot}21\,\delta V.$$

If, therefore, this result be equated to the assumed correction required by Hansen's value of the coefficient of the parallactic inequality, instead of obtaining the true equation

$$1{\cdot}90\,\delta P = D,$$

we shall obtain the expression

$$1{\cdot}90\,\delta P = D + 0{\cdot}21\,\delta V.$$

The resulting value of $\delta P$ will be, therefore,

$$\delta P = \frac{10}{19} D + \frac{1}{9}\delta V,$$

or will be too large by one ninth of the correction required by the coefficient of the variation. If, therefore, $\delta[P]$ be the value found for the correction to the coefficient of the parallactic inequality where the term in $\delta V$ is neglected, the true value will be

$$\delta P = \delta[P] - \frac{1}{9}\delta V.$$

Now, Burckhardt's value of the variation is greater than Hansen's by

$$\delta V = 3''{\cdot}85,$$

this being the sum of the correction to the variation and to some analogous terms which will act on the whole like an apparent increase to the value of the variation. Consequently, the value found from these observations for the correction to the parallactic inequality must be decreased to the amount

$$\delta P = \delta[P] - \frac{3''{\cdot}85}{9} = \delta[P] - 0''{\cdot}43.$$

The importance of this result is evident, for it shows that the results deduced for the correction to the parallactic inequality from the comparison of Hansen's tables with the Greenwich Observations for the years 1847–1858 are all affected by a systematic error which will render the apparent correction erroneous by nearly half a second of arc. But these observations form three fifths of the data employed by Mr. Stone, so that his result will be affected by a proportionately large error.

The only remaining term to be considered is that with the argument $3\theta$. The important effects of an error in this term

have been already shown, and its amount approximately determined. From a comparison of Hansen's and Delaunay's values for Q, the coefficient of this term, with that made use of by Airy, it appears that Airy has adopted too large a value. Delaunay's value is $+ 0''.53$ but an examination of the algebraical form of his coefficient shows that he has not pushed his approximation far enough, and, as every additional term tends to reduce its value, it is probable that his value is too great by $0''.07$ at least. This would reduce it to $+ 0''.46$. Neison's value of the coefficient is $+0''.44$, but this extends an order further than Delaunay's. Hansen's value is $+ 0''.41$, and is probably very nearly correct. Comparing these values with Airy's, it is evident that Airy's coefficient is too large by

$$\delta Q = +0''.49.$$

Consequently, the value found by the Astronomer Royal for the value of the parallactic inequality ought to be increased by

$$\frac{1}{3}\delta Q = +0''.16.$$

On comparing the theoretical value of this coefficient with that employed by Burckhardt, which was $2''.7$, it appears that Burckhardt's value is too great by

$$\delta Q = +2''.29.$$

Consequently, if Mr. Stone had determined the parallactic inequality from the observations made at Greenwich in the years 1859, 1860, 1861, as these are compared with Burckhardt's tables, the resulting value of the parallactic inequality would require to be increased by

$$\frac{3}{4}\delta Q = +1'.71.$$

When the term with the argument $3\theta$ is examined, it will be found to exert a perfectly similar effect on the comparison between Hansen's and Burckhardt's tables, to that already shown to occur with the variation. There is this distinction, however, that whereas in the case of the variation the effect before and after the Quarter had the same sign, in the case of this term with the argument $3\theta$ they will differ in sign and so tend to neutralise each other. If, therefore, the number of observations before the day of first Quarter or after that of third Quarter were nearly equal to those made after the first Quarter and before the third Quarter, the errors from this source would counterbalance one another. This, however, is not the case, as the latter class of observations far exceed in number the former, consequently some considerable portion of the correction will remain outstanding. By the same method which was employed in the case of the variation, the effect may be shown to be represented by the expression

$$\delta P = \delta[P] - \frac{1}{25}\delta Q.$$

Substituting the value found for $\delta Q$,

$$\delta P = \delta[P] - 0''.09.$$

The result, therefore, of this examination may be summed up as follows :—

Airy's value requires a correction of

$$+0''.16.$$

The portion of Stone's results which are based on the comparison between Hansen's tables and the Greenwich Observations for the years 1848 to 1858 require a correction of

$$-0''.52.$$

The portion, if any, of Stone's results founded on the Greenwich Observations of 1859, 1860 and 1861, requires the correction

$$+1''.71.$$

Both Stone's and Newcomb's results will require correction, if there be any error in the tabular value of the coefficient of the variation employed in Hansen's tables.

The results obtained by Sir G. Airy (*Memoirs Roy. Ast. Soc.* vol. xxix. 1–24) are founded on an elaborate discussion of the entire series of Greenwich Observations made between 1750 and 1851. The observations are divided into nine-year groups, and from each of these groups there are simultaneously deduced corrections to the epoch, eccentricity, motion of the perigee, and coefficients of the evection, annual equation, variation, and parallactic inequality. Therefore, the influence of errors in the tabular values of all these terms is eliminated from the resulting value of the parallactic inequality. The early groups can be omitted, as they must be inferior in accuracy to the more modern observations. The remaining nine groups yield the following results.

| Group. | Mean Year. | Correction to the Assumed Coefficient. | Resulting Coefficient of the Parallactic Inequality. | Difference from Mean. |
|---|---|---|---|---|
| | | ′ | ′ | ′ |
| 1806–1815 | 1810·5 | −0·88 | −122·98 | +1·24 |
| 1811–1819 | 1815·0 | −1·99 | −124·09 | +0·13 |
| 1816–1824 | 1820·0 | −2·40 | −124·50 | −0·28 |
| 1820–1829 | 1824·5 | −3·72 | −125·82 | −1·60 |
| 1825–1833 | 1829·0 | −3·10 | −125·20 | −0·98 |
| 1830–1838 | 1834·0 | −1·91 | −124·01 | +0·21 |
| 1834–1842 | 1838·0 | −1·47 | −123·57 | +0·65 |
| 1839–1847 | 1843·0 | −1·14 | −123·24 | +0·98 |
| 1843–1851 | 1847·0 | −2·43 | −124·53 | −0·31 |

It will be seen from the last column that there is a considerable discordance between the separate results for each group of years, but, treating these discordances as due to accidental or unsystematic errors, the value found for the coefficient of the parallactic inequality is

$$(P) = -124''.22 \pm 0''.20.$$

The mean of the differences of the separate results from the mean of all the results is $0''.71$, a very large quantity. The Astronomer Royal, by omitting the first of these groups, has found the value

$$(P) = -124''.37.$$

The coefficient whose value has been determined by Sir G. Airy is not the real coefficient of the parallactic inequality, and which can be alone used for determining the distance of the Sun, but it is the apparent coefficient of the parallactic inequality, or the sum of the true coefficient *plus* the factor which expresses the influence of the variation in the semi-diameter. No attempt was made by the Astronomer Royal to determine the effect of this variation in semi-diameter, so that, unless this be subsequently done by a further discussion of the observations, it will be impracticable to determine the real coefficient of the parallactic inequality from these investigations, or to use them for ascertaining the solar parallax.

The periodical character of the differences between the mean value and the separate results is obvious, the values obtained for the parallactic inequality at different times seeming to be affected by a periodical inequality running through all its changes in about 30 years and reaching its maximum value between the years 1825 and 1826. It has already been shown that this would indicate the existence of some imperfection in the lunar tables due to the omission of a term with a period differing slightly from that of the parallactic inequality. Suppose, therefore, it be assumed that

$$-X = \delta(P) + B\cos b,$$

where $\delta(P)$ denotes the true correction to the apparent coefficient of the parallactic inequality, and B cos $b$ denotes a periodical inequality in X, the values obtained from any group of observations for the value of the apparent coefficient of the parallactic inequality. As a first approximation it may be assumed that

$$b = 12^{\circ} \times (Y - 1825{\cdot}5).$$

Then the nine values of X obtained by Sir G. Airy from the discussion of the Greenwich Observations will give nine equations from which to determine the values of $\delta(P)$ and B. Solving these equations by the method of least squares, they give the values

$$\delta(P) = +2''.37, \qquad B = +1'.18.$$

The comparison of the correction calculated from these values with the correction actually obtained from the observations is as follows :—

| Epoch | Observed Correction. | Difference from Mean. | Calculated Correction. | Difference from Observed. |
|---|---|---|---|---|
| 1810·5 | −0″·88 | +1″·24 | −1″·18 | +0″·30 |
| 1815·0 | −1·99 | +0·13 | −1·67 | +0·32 |
| 1820·0 | −2·40 | −0·28 | −2·85 | −0·45 |
| 1824·5 | −3·72 | −1·60 | −3·49 | −0·23 |
| 1829·0 | −3·10 | −0·98 | −3·25 | +0·15 |
| 1834·0 | −1·91 | +0·21 | −2·11 | +0·20 |
| 1838·0 | −1·47 | +0·65 | −1·34 | −0·13 |
| 1843·0 | −1·14 | +0·98 | −1·34 | +0·20 |
| 1847·0 | −2·43 | −0·31 | −2·11 | −0·32 |

The smallness of the residuals shows how closely the hypothesis of a missing term accounts for the variation in the values found by the Astronomer Royal for the apparent coefficient of the parallactic inequality. If the hypothesis which has been made is correct, then the term missing from the tables would have the form

$$+1''\cdot18 \sin \{\theta + 12^\circ \times [Y - 1825\cdot5]\},$$

or it may be

$$+1\cdot18 \sin \{(a-\mu) + 52^\circ\cdot7 \times [Y - 1819\cdot5]\}.$$

The weakest point in the form assigned to the preceding inequality is its period. It is obvious that a term of longer period would satisfy all the observations except the group for 1843—1851, or rather that portion of the group included between the years 1847—1851. When this group is reached the inequality seems to suddenly change sign, thus limiting its period to thirty years. It is thus solely this group which suggests the adoption of a thirty-year inequality.

When the results for the earlier groups of observations, those made prior to 1800, are compared with the values found by the preceding hypothesis, the comparison is far from being so satisfactory and reveals much larger discordances. It is true that these earlier observations are much less precise than the more modern ones, so that they might fairly be expected to show larger discordances. These earlier values were obviously affected by an analogous inequality to the more modern, but the variations indicated an inequality of longer period than thirty years. These earlier groups gave corrections to the assumed coefficient of the parallactic inequality which rendered it more than a second smaller than the value indicated by the more modern groups. This constant difference, however, is no more than might be expected, for the instruments employed in the two periods

were different, and the earlier and smaller instrument would certainly give a smaller value for the *apparent* coefficient of the parallactic inequality than would be the case with the finer modern instrument, as in the earlier instrument there would be a greater variation in the irradiation of the lunar limb, and consequently a greater decrease in the apparent value of the coefficient of the inequality.

It being only the final group for the period 1847–1851 which prevented a longer period being assigned to the inequality indicated by the more modern groups of observations, the group depending on these observations was omitted, it being the final one of the series, and the remaining twenty values of the correction to the parallactic inequality were considered. They gave the results for the ninety-seven years 1750–1846. All the earlier observations were reduced by Sir G. Airy with the same value for the mean semi-diameter of the Moon. It was probable, however, that more accurate results would be obtained by using for each group of observations the value of the apparent semi-diameter given by the observations made during the period covered by the group. Moreover, by adopting this system the greater part of the effects of any personality would be eliminated. The data requisite to effect this are given in the Introduction to the *Reduction of the Greenwich Lunar Observations* (vol. i. pages lxiv.–lxv.). From these data corrections were obtained to the value of the semi-diameter employed by Sir G. Airy in reducing the observations contained in each group between 1750 and 1816. From these corrections it appeared that the observations had been reduced with too large a semi-diameter from 1800 to 1775, and with too small a value from 1775 to 1750. The resulting corrections to the value found for the apparent coefficient of the parallactic inequality varied between $+ 0''\cdot70$ for the year 1755 to $- 1''\cdot20$ for the year 1780. The application of these corrections rendered still more apparent the regular periodical variation in the values found for the correction to the coefficient of the parallactic inequality.

As before, let $\delta(P)$ denote the correction to the parallactic inequality, B denote the coefficient of the forty-five-year inequality, and A denote the constant correction required by the earlier observations; then the twenty values for the apparent correction to the coefficient of the parallactic inequality furnish twenty equations from which to determine $\delta(P)$, B, and A. Solving these by the method of least squares, the following values are obtained :—

$$\delta(P) = +1''\cdot85, \qquad B = +1''\cdot20, \qquad A = +1''\cdot60.$$

From these values was calculated the theoretical apparent coefficient of the parallactic inequality for each of these groups of observations, and it was compared with the observed value.

| Epoch. | Observed Coefficient. | Calculated Coefficient. | Difference. | Epoch. | Observed Coefficient. | Calculated Coefficient. | Difference. |
|---|---|---|---|---|---|---|---|
| 1843·0 | −123·24 | −123·00 | −·24 | 1796·5 | −121·34 | −121·56 | +·22 |
| 1838·0 | −123·57 | −123·67 | +·10 | 1792·0 | −121·95 | −122·15 | +·20 |
| 1834·0 | −124·01 | −124·29 | +·28 | 1787·0 | −122·26 | −122·92 | +·66 |
| 1829·5 | −125·20 | −124·95 | −·25 | 1783·0 | −123·56 | −123·41 | −·15 |
| 1824·0 | −125·82 | −125·50 | −·32 | 1778·0 | −123·93 | −123·42 | −·51 |
| 1820·0 | −124·50 | −124·85 | +·35 | 1773·5 | −122·54 | −122·99 | +·45 |
| 1815·0 | −124·09 | −124·15 | +·06 | 1769·0 | −122·10 | −122·32 | +·22 |
| 1810·5 | −122·63 | −122·60 | −·03 | 1764·0 | −122·03 | −121·56 | −·47 |
| 1806·0 | −121·46 | −121·25 | −·21 | 1759·5 | −121·00 | −121·14 | +·14 |
| 1801·0 | −120·82 | −121·13 | +·31 | 1754·5 | −120·73 | −121·20 | +·47 |

The small residuals between the calculated and observed places, although the observations extend over more than a double revolution of the inequality, show how strongly the existence of this inequality is indicated by the observations. Small as the residuals are, they could be still further reduced by slightly altering the period and changing by a small interval the time when the inequality is supposed to reach its maximum value.

The computed value of the correction for the group 1843–1851 is (P)$=-122''\cdot75$, whilst that actually found from the observations is (P)$=-124''\cdot53$, there being a difference of $1''\cdot78$. Almost the entire portion of this difference arises from the high values found for the parallactic inequality from the observations made during the years 1846–1851, as those made for the earlier years were much lower. It would, in fact, seem as if the apparent semi-diameter of the Moon had increased by $2''\cdot0$ during these years, either from a change of observers or some other cause. This would have increased the apparent value of the parallactic inequality by $2''\cdot6$, and as this increase affects two thirds of the observations, it would correspond to a fictitious increase of $-1''\cdot73$, whilst that actually observed was $-1''\cdot78$. The further consideration of this point must be deferred.

Sir G. Airy obtained the value for the apparent coefficient of the parallactic inequality of

$$(P) = -124''\cdot37.$$

Neglecting the effects of the supposed inequality, but increasing the earlier observations by the constant factor A$=1''\cdot60$ to allow for the difference in instruments, the value becomes

$$(P) = -123\cdot78.$$

Upon the hypothesis of a thirty-year inequality, the value is

$$(P) = -124\cdot47.$$

Upon the more probable hypothesis of a forty-five-year inequality, the value of (P) becomes

$$(P) = -123''.95.$$

This last value seems to be entitled to far greater weight than any of the others, and is probably close to the truth. It involves the assumption of the real existence of the forty-five-year inequality, a question whose further consideration must be deferred for the present.

Since the above results were completely finished, a reference to the earlier memoir by Sir G. Airy (*Memoirs Roy. Ast. Soc.*, vol. xvii. page 42) shows that the periodical nature of the variations in the values found for this inequality did not escape his attention. The Astronomer Royal remarks :—"We might almost imagine, from the succession of values of V, that it is subject to a periodical factor whose period is about 46 years. Such a factor would be represented by adding to the Moon's longitude two inequalities whose daily increases of argument are $13^s.5686$ and $13^s.5220$. I cannot conceive that there is any physical foundation for this supposition." Is the existence of such inequalities inconceivable?

From a discussion of the four years' observations made with the Altazimuth between 1847 and 1851, the Astronomer Royal derived the value

$$(P) = -125''.50.$$

This excessive value he seemed disposed to regard as accidental. This, however, is by no means the case. The Astronomer Royal has derived the supposed differences between his own tables and observation, through the medium of Burckhardt's tables, in exactly the same manner as he derived the supposed differences between Hansen's tables and observation for the years 1847–1858. It has been already shown that this will introduce grave systematic errors. In this case these errors will require the value obtained by Sir G. Airy to be reduced by $-0''.46$, as a greater portion of the error in the term $3\theta$ will be eliminated. Further, in reducing the Altazimuth observations Sir G. Airy has employed the same semi-diameter as in reducing the Meridian observations, whereas the former seems to require a semi-diameter at least $0''.6$ greater, and for this reason the Astronomer Royal's value will be about $0''.75$ too large. Applying these corrections, its value becomes

$$(P) = -124''.29.$$

Much weight cannot, however, be given to this result, from the uncertainty as to the exact difference in semi-diameter between the Altazimuth and old Transit Instrument (the difference between the Altazimuth and Transit Circle is known to be fully

1″·0), and partly from the small period over which the observations extend.

All the values in the preceding statement require the correction −0″·16 for the errors in Airy's tables. The value found by the Astronomer Royal from the modern observations should be

$$(P) = -124''.53,$$

and the value found from all the observations by the assumption of a forty-five-year inequality becomes

$$(P) = -124''.11.$$

All these values are those of the apparent coefficient of the parallactic inequality, and are therefore in their present state useless for the purpose of yielding the solar parallax. Nor are they strictly comparable in this form with the results obtained from other instruments, for, being affected by the error due to the variation in the irradiation at the limb, they are affected by an error which varies with the aperture and excellence of the telescope.

Prof. Hansen does not seem to have published any details of the method by which he determined his value of the parallactic inequality, so that there is no means of ascertaining in what manner he eliminated the effect of the variation in the semidiameter of the Moon. Practically the only existing information seems to be contained in the Note in the *Monthly Notices* for November 1854 (vol. xv. page 9). He states there that from a comparison of his own theory with some of the Greenwich observations, and some of the Dorpat observations of the Moon, he deduced as the value of the coefficient of the parallactic inequality

$$P = -126''.46.$$

It appears to be probable that the observations employed by Prof. Hansen were made between the years 1820 and 1835, for it is certain that he did compare many of the Greenwich observations made at this period with his tables. It is noteworthy that at this period the Greenwich observations give the largest value for the apparent coefficient of the parallactic inequality, a value more than 1″ greater than what is probably the true value, and Hansen's value is in turn apparently 1′ too large. Unless, however, the papers of Prof. Hansen which are now at Leipzic are found to afford some further information on the subject, nothing can be done with this result obtained by Prof. Hansen.

The only published account of Mr. Stone's investigation is to be found in a short abstract of his memoir published in the *Monthly Notices* for May 1867 (vol. xxvii. page 271). The memoir

itself seems never to have been published, and is not now to be found. In consequence of this misfortune, the details of the method which was followed have to be surmised from the very short abstract of the Introduction, which was all that was ever published. The main principle of the method adopted is stated to have been as follows. The longitude deduced from the observations made near the maximum of the parallactic inequality was compared with the tabular longitude; and (it being assumed that the error in the mean results arises entirely from error in the coefficient of the parallactic inequality) the comparison furnishes a correction to the coefficient in question, and consequently also to the value of the solar parallax used in the calculation of that coefficient. It is further stated that 2,075 observations were used, all made at Greenwich between 1848 and 1866. The value obtained for the coefficient was

$$(P) = -125''.36 \pm 0''.40.$$

This has been put down as the apparent coefficient because, if the preceding description fully describes what was done, it is obvious that no steps were taken to eliminate the effect of the variation in the semi-diameter due to differences in contrast between the sky and the limb of the Moon. If this be so, then it is obvious that Prof. Newcomb in using it as the value of the real coefficient of the parallactic inequality has been misled. Prof. Newcomb lays great stress on the importance of distinguishing between the two, and his own investigations show that the apparent value of the inequality is less than the real value by over 1″. Consequently, had he perceived that the value obtained by Mr. Stone is really that of the apparent coefficient and not that of the real coefficient, he would have seen that, so far from confirming his own results, there was a discordance of fully 1″.

From the number of observations used, which is stated to be 2,075, or at the rate of at least 109 a year, it is obvious that both the Meridian and Altazimuth observations were used. Although not explicitly stated, it is probable that the actual figures used were those for the years 1848—1858 given as the comparison between the Greenwich Observations and Hansen's tables in the Appendix to the Greenwich Observations in the volume for 1859 and those for the years 1862—1866 given in the annual volumes for these years. All these may be said to be founded on Hansen's tables. It is uncertain whether the observations made at Greenwich during the years 1859, 1860, 1861 were used, as they are compared with Burckhardt's tables. No information exists as to the value employed for the semi-diameter, but it is probable that Hansen's tabular value was used unchanged. This is, in fact, implied in the sentence in brackets, that the mean difference between observation and theory was supposed to arise entirely from error in the coefficient of the parallactic inequality. For

the same reason it is probable that the same semi-diameter was used for both Meridian and Altazimuth observations.

The apparent semi-diameter of the Moon with the Altazimuth is fully 1″ greater than with the Transit Circle so that the parallactic inequality derived from the Altazimuth observations will be over 1″ greater than that derived from the Transit Circle. Of late years the observations with the Altazimuth have been reduced with a large value of semi-diameter, thus reducing the error; but probably the principal observations made use of by Mr. Stone would not be corrected for the difference in semi-diameter. On this account, as one half of the observations employed by Mr. Stone are affected by this correction, his result would require to be reduced by 0″·50.

It has already been shown that the results derived from the observations made during the years 1848–1858 ought to be decreased by −0″·52. As these results affect three-fifths of the observations employed by Mr. Stone, his result will require a correction of −0″·31. It will be, therefore,

$$(P) = -125''.05.$$

If this be further reduced by the correction for the difference in semi-diameter required by the Altazimuth and Transit Circle observations, it will become

$$(P) = -124''.55.$$

The investigation by Prof. Newcomb is contained in Appendix II. to the Washington Observations for 1865. In this memoir a sufficiently full account is given of the method by which the value of the parallactic inequality was deduced, so that its real character can be properly made out. It is founded on the Meridian observations of the Moon made at Washington during the four years 1862, 1863, 1864 and 1865. The number of observations which were used probably did not exceed one hundred.

According to Prof. Newcomb, the method employed by him to determine the value of (P), the apparent coefficient of the parallactic inequality, was as follows: "The Washington Observations of the Moon, from 1862 to 1865 inclusive, are regularly compared with Hansen's tables. I have discussed those made within two days of the time of maximum and minimum parallactic inequality, on the supposition that the effect of errors in the other inequalities will destroy each other in the course of the four years. Thus the following corrections to Hansen's parallactic inequality are obtained for the several years:—

$$1862 = +2''.2 \qquad 1863 = +2''.2 \qquad 1864 = +2''.0 \qquad 1865 = +2''.0.$$

These results are still subject to correction for adopted semi-

diameter of the Moon. Seven transits of both limbs of the nearly Full Moon were observed during the above period. The mean correction to Hansen's semi-diameter was zero. If, then, we suppose this same semi-diameter applicable to the Moon at first and third Quarters, the coefficient of the parallactic inequality will be"

$$(P) = -126''.46 + 2''.10 = -124''.36.$$

Prof. Newcomb next considers how to convert this, the value for the apparent coefficient, into the true value of the coefficient. He points out that as many of the observations at the Quarters are made in bright twilight, or even daylight, from the diminished irradiation the apparent semi-diameter of the Moon must be considerably smaller than at Full, and that consequently the above value of the parallactic inequality must be increased to make it correspond to the real value of the parallactic inequality.

After a careful discussion of the probable amount of difference between the semi-diameter at Full Moon and at its quarters when observed at daylight, Prof. Newcomb arrives at the conclusion that from this variation in the semi-diameter the real value of the parallactic inequality will exceed the apparent value by $1''.10$. In this manner he deduces for the real value

$$P = -124''.36 - 1''.10 = -125''.46.$$

As Prof. Newcomb points out, it is the uncertainty as to the real value of this augmentation due to the variation of the lunar semi-diameter which forms the whole difficulty in the problem. The value obtained by him can be scarcely said to rest on observation.

Prof. Newcomb regards his value as in perfect accord with that obtained by Mr. Stone, for the result of the Greenwich observations. We have seen that this is a mistake, and that Mr. Stone's is really the value of the apparent coefficient and corresponds really to the value $124''.36$ obtained by Prof. Newcomb.

It may here be remarked that both Mr. Stone and Prof. Newcomb expressly base their investigations on the assumption, that in the mean result the errors in any other of the inequalities will destroy each other. This assumption it has been shown is untrue for at least three inequalities in the motion of the Moon, and that any error in Hansen's values for the annual equation or variation will be in part thrown on the values found by them for the coefficient of the parallactic inequality. These errors may be very approximately taken as

$$= +\frac{1}{9}\delta V - \frac{1}{7}\delta M.$$

Moreover, if there really exists an inequality of forty-five-year period in the apparent value of the parallactic inequality,

and if, further, this inequality does not arise from imperfections in Airy's tables which do not exist in Hansen's tables, then all these values will be affected by this inequality. Introducing then all these elements of uncertainty, the results can be written in the form

$$\text{Airy} \qquad (P) = -124''.53 + \frac{1}{3}B,$$

$$\text{Stone} \qquad (P) = -124''.55 - \frac{1}{7}B + \frac{1}{9}\delta V - \frac{1}{7}\delta M,$$

$$\text{Newcomb} \quad (P) = -124''.36 + \frac{2}{5}B + \frac{1}{9}\delta V - \frac{1}{7}\delta M.$$

These results are those which, we believe, really represent the true values of the apparent coefficient of the parallactic inequality to be derived from the investigation of these astronomers when properly corrected. A comparison of these results with those originally given will show the importance of the correction which we have been obliged to introduce, and the necessity which existed for a rigid examination of the theoretical basis of their labours.

[The concluding portion of this Paper, containing a new discussion of the Greenwich Meridian observations of the Moon in the years 1862–1876, will be published in the June No.—Ed.]

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*Note on the Astronomer Royal's Investigation of the Theoretical Value of the Acceleration of the Moon's Mean Motion.* By Professor J. C. Adams, M.A., &c.

I lose no time in pointing out briefly the reason why the Astronomer Royal, in the investigation which he communicated to the Society at the last Meeting, has failed to find my value of the coefficient of the Lunar Acceleration.

It may be useful, in the first place, to recall to mind that, according to my theory, the secular changes of

$n$, the Moon's mean motion,

and $e'$, the eccentricity of the Earth's orbit,

are connected by the following relation:—

$$\frac{dn}{ndt} = \frac{e'de'}{dt}\left\{-3m^2 + \frac{3771}{32}m^4 + \frac{34047}{42}m^5 + \ldots\right\},$$

where $m$ denotes, as usual, the ratio of the Sun's mean motion to that of the Moon.

If we stop at the first term of the series within the brackets the result is identical with that found by Laplace.

We do not know why Laplace did not carry his investigations further than this first term; but he probably thought that the succeeding terms would prove to be inconsiderable.

It is seen, however, that these terms have very large numerical coefficients and that their sign is contrary to that of the first term, and on calculation it is found that the sum of the series is less than its first term nearly in the ratio of 3 to 5.

Hence the secular acceleration will be diminished in the same ratio, and its amount in a century, instead of being about 10″, will be reduced to nearly 6″.

No investigation of the Moon's secular acceleration can be satisfactory which does not take into account terms of the nature of those which give rise to the terms involving $m^4$, $m^5$, &c., above referred to.

There is nothing to object to in the general principles of the method adopted by the Astronomer Royal, but in the practical application of the method I notice very grave defects.

In the first place, the only periodic terms which are included in the Astronomer Royal's expressions for $\mathrm{T}\frac{r}{a}$ and $\mathrm{P}\frac{r}{a}$ and for the factors multiplying

$$\delta\frac{a}{r},\quad \frac{d}{dt}\left(\delta\frac{a}{r}\right),\quad \delta v,\quad \frac{d}{dt}(\delta v),\quad \&c.,$$

on the right-hand side of the equations, are those which involve the angle 2D or F; whereas it will be seen by a reference to my paper in the *Philosophical Transactions* for 1853, that a great part of the coefficient of $m^4$ in the value of $\frac{dn}{ndt}$ there obtained arises from the combination of terms involving the angles S, F−S and F+S in the expressions for the Moon's coordinates with similar terms in

$$\delta\left(\frac{a}{r}\right),\quad \delta v,\quad \&c.$$

In the present investigation terms of the forms last mentioned are simply ignored.

In the next place, it is to be noted that, although periodic terms depending on the angle F are introduced into the assumed values of $\delta\frac{a}{r}$ and $\delta v$, yet in art. 12, the value of $h$ which is the coefficient of $t^2$ in the value of $\delta v$, is found equal to $-\mathrm{B}b$, quite independently of the values of the coefficients $e$, $f$, $g$, $k$ and $l$, which occur in the terms thus introduced.

The result of this is to reduce the secular acceleration practically to its first term only; which accounts for the coincidence of the Astronomer Royal's value with that of Laplace.

It may also be remarked in reference to art. 11, that although

terms involving the argument 2F or 4D may be properly omitted, we must put

$$\sin^2 F = \frac{1}{2} - \frac{1}{2}\cos 2F,$$

$$\text{and } \cos^2 F = \frac{1}{2} + \frac{1}{2}\cos 2F,$$

and the constant terms in these latter quantities should be taken into account.

After these general remarks, we will enter a little more closely on the consideration of one or two points in the investigation which are important.

Adopting the Astronomer Royal's notation, let

$\sigma$ denote the Sun's mass,

A the semiaxis major of the Sun's (or Earth's) orbit,

E the eccentricity of the orbit,

R the radius vector at any time.

Then it may be shown, as in the paper before us, that the mean value of

$$\frac{\sigma}{R^3} \text{ is } \frac{\sigma}{A^3}\frac{1}{(1-E^2)^{\frac{3}{2}}} = \frac{\sigma}{A^3}\left(1+\frac{3}{2}E^2\right) \text{ nearly.}$$

Hence if E receive the variation $\delta E$ in the time $t$, this quantity will be increased in the ratio of $1+3E\delta E$ to 1 nearly, or in the ratio of $1+bt$ to 1, calling

$$3\frac{E\delta E}{t} = b.$$

Having arrived at this point, the Astronomer Royal assumes that the variation of the disturbing forces due to the variation $\delta E$ in the eccentricity of the Sun's orbit will be represented by supposing

$$T \text{ to be replaced by } T(1+bt),$$

$$\text{and similarly P to be replaced by } P(1+bt),$$

and therefore that the new forces, the effects of which are to be found by the present method, are $Tbt$ and $Pbt$ respectively.

On consideration, however, it will appear that this is only true for the non-periodic term in P, and that the periodic terms, whether in P or T, will be changed by any given variation of E in very different ratios.

For instance, the periodic terms in both T and P which depend on the angle 2D or F will vary nearly in the same ratio as $1-\frac{5}{2}E^2$ does, instead of in the ratio in which $1+\frac{3}{2}E^2$ varies as in the above case.

Hence these terms will be changed by the above-mentioned variation of E in the ratio of $1 + b't$ to 1, where

$$b' = -5\frac{E\delta E}{t} \text{ nearly.}$$

Again, the periodic terms in T and P which depend on the angles S, F−S and F+S will vary nearly in the same ratio as E does, so that these terms will be changed in the ratio of $1 + b''t$ to 1, where

$$b'' = \frac{\delta E}{Et} \text{ nearly.}$$

Hence we see that the values of $b'$ and $b''$ are quite different from that of $b$ which belongs to the non-periodic term, and that $b''$ is much larger than the other two quantities.

The correct way of finding $\delta T$ and $\delta P$, the changes of the disturbing forces T and P due to change in the eccentricity of the Sun's orbit, is to express T and P in terms of the Moon's coordinates $v$ and $r$, the Sun's mean longitude L and its mean anomaly S, and the eccentricity E.

Hence $\delta T$ and $\delta P$ may be at once expressed in terms of $\delta v$, $\delta r$, and $\delta E$.

Thus calling V the Sun's longitude, and employing the other symbols in the sense before explained, we have

$$P = \frac{1}{2}\frac{\sigma r}{R^3} + \frac{3}{2}\frac{\sigma r}{R^3}\cos(2\nu - 2V),$$

$$T = -\frac{3}{2}\frac{\sigma r}{R^3}\sin(2\nu - 2V).$$

Or,

$$Pr = \frac{1}{2}\frac{\sigma r^2}{R^3} + \frac{3}{2}\frac{\sigma r^2}{R^3}\cos(2\nu - 2V),$$

$$Tr = -\frac{3}{2}\frac{\sigma r^2}{R^3}\sin(2\nu - 2V).$$

Now, by the formulæ of elliptic motion, we may find

$$\frac{1}{R^3} = \frac{1}{A^3}\left[1 + \frac{3}{2}E^2 + 3E\cos S\right],$$

$$\frac{1}{R^3}\cos(2\nu - 2V) =$$

$$\frac{1}{A^3}\left\{\left(1 - \frac{5}{2}E^2\right)\cos(2\nu - 2L) + \frac{7}{2}E\cos(2\nu - 2L - S) - \frac{1}{2}E\cos(2\nu - 2L + S)\right\},$$

$$\frac{1}{R^3}\sin(2\nu - 2V) =$$

$$\frac{1}{A^3}\left\{\left(1 - \frac{5}{2}E^2\right)\sin(2\nu - 2L) + \frac{7}{2}E\sin(2\nu - 2L - S) - \frac{1}{2}E\sin(2\nu - 2L + S)\right\},$$

neglecting terms involving 2S, and powers of E above the second.

Substituting, and then taking the variation, we have

$$\delta(\mathrm{P}r) = \frac{\sigma}{\mathrm{R}^3} r\delta r + 3\frac{\sigma}{\mathrm{R}^3} r\delta r \cos(2\nu - 2\mathrm{V}) - 3\frac{\sigma}{\mathrm{R}^3} r^2\delta\nu \sin(2\nu - 2\mathrm{V})$$

$$+\frac{1}{2}\frac{\sigma r^2}{\mathrm{A}^3}\Big[3\mathrm{E}\delta\mathrm{E} + 3\delta\mathrm{E}\cos\mathrm{S}\Big]$$

$$+\frac{3}{2}\frac{\sigma r^2}{\mathrm{A}^3}\Big[-5\mathrm{E}\delta\mathrm{E}\cos(2\nu - 2\mathrm{L}) + \frac{7}{2}\delta\mathrm{E}\cos(2\nu - 2\mathrm{L} - \mathrm{S})$$

$$-\frac{1}{2}\delta\mathrm{E}\cos(2\nu - 2\mathrm{L} + \mathrm{S})\Big]$$

$$\delta(\mathrm{T}r) = -3\frac{\sigma}{\mathrm{R}^3} r\delta r \sin(2\nu - 2\mathrm{V}) - 3\frac{\sigma}{\mathrm{R}^3} r^2\delta\nu \cos(2\nu - 2\mathrm{V})$$

$$-\frac{3}{2}\frac{\sigma r^2}{\mathrm{A}^3}\Big[-5\mathrm{E}\,\delta\mathrm{E}\sin(2\nu - 2\mathrm{L}) + \frac{7}{2}\delta\mathrm{E}\sin(2\nu - 2\mathrm{L} - \mathrm{S})$$

$$-\frac{1}{2}\delta\mathrm{E}\sin(2\nu - 2\mathrm{L} + \mathrm{S})\Big]$$

in which $-r^3\delta\left(\frac{1}{r}\right)$ may be written for $r\delta r$, and the expressions given by the ordinary lunar theory in the case of unvaried eccentricity are to be substituted for $v$ and $r$.

Hence, the expressions for $\delta\left(\mathrm{T}\frac{r}{a}\right)$ and $\delta\left(\mathrm{P}\frac{r}{a}\right)$, which are employed in the paper, are wholly incorrect, except in the case of the non-periodic term, which gives rise to the principal term of the secular acceleration or that found by Laplace.

The remark made near the close of the paper, viz. that the magnitudes of the quantities A, B, C, and therefore also that of the secular acceleration are proportional to the inverse cube of the Sun's distance, or to the cube of the Sun's parallax, can only be the result of inadvertence, as the Astronomer Royal himself will be the first to acknowledge.

In fact, the quantities A, B, C involve the factor $\frac{\sigma}{\mathrm{A}^3}$ and this is equal to $n'^2$, where $n'$ is the Sun's mean motion and is known. The Sun's mass $\sigma$ is determined by means of the parallax from this equation; or conversely, if the Sun's mass be known the parallax is thereby determined.

The values of A, B, C are approximately as follows

$$\mathrm{A} = \frac{3}{2}m^2, \quad \mathrm{B} = \frac{1}{2}m^2, \quad \mathrm{C} = \frac{3}{2}m^2,$$

where $m$ denotes, as before, the ratio of the Sun's mean motion to that of the Moon.

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Errata in *Monthly Notices*, vol. xix. p. 207, l. 6, for $-3m^4$ read $-3m^2$.

p. 208, l. 15 for $\frac{dn}{n^2dt}$ read $\frac{dn}{ndt}$.

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## *Ephemeris for Physical Observations of Jupiter*, 1880–81. By A. Marth, Esq.

| Greenw. Noon. | Angle of Position of ♃'s Axis. | Latitude of Earth above ♃'s Equator. | Latitude of Sun above ♃'s Equator. | Annual Parallax. | Equat. Diam. | Greatest Phase. | Longitude of ♃'s Meridian directed to the Earth. | Diff. | Corr. of Long. |
|---|---|---|---|---|---|---|---|---|---|
| 1880. | ° | ° | ° | ° | ″ | ″ | ° | ° | ° |
| June 18 | 335·698 | +2·616 | +2·322 | −11·26 | 37·83 | 0·364 | 196·69 | | +0·55 |
| 23 | 335·792 | 2·655 | 2·338 | 11·50 | 38·38 | ·385 | 229·37 | 4352·68 | ·57 |
| 28 | 335·879 | 2·693 | 2·353 | 11·68 | 38·95 | ·403 | 262·11 | 2·74 | ·59 |
| July 3 | 335·959 | 2·730 | 2·369 | 11·80 | 39·55 | ·418 | 294·91 | 2·80 | ·61 |
| 8 | 336·032 | 2·766 | 2·384 | 11·86 | 40·17 | ·429 | 327·78 | 2·87 | ·61 |
| 13 | 336·096 | 2·800 | 2·400 | 11·84 | 40·82 | ·434 | 0·72 | 2·94 | ·61 |
| 18 | 336·152 | 2·832 | 2·415 | 11·76 | 41·48 | ·435 | 33·73 | 3·01 | ·60 |
| 23 | 336·198 | 2·863 | 2·430 | 11·60 | 42·15 | ·430 | 66·81 | 3·08 | ·59 |
| 28 | 336·232 | 2·892 | 2·445 | 11·36 | 42·84 | ·420 | 99·97 | 3·16 | ·56 |
| Aug. 2 | 336·255 | 2·919 | 2·460 | 11·04 | 43·53 | ·403 | 133·19 | 3·22 | ·53 |
| 7 | 336·265 | 2·944 | 2·474 | 10·64 | 44·22 | ·380 | 166·48 | 3·29 | ·49 |
| 12 | 336·262 | 2·966 | 2·489 | 10·16 | 44·90 | ·352 | 199·84 | 3·36 | ·45 |
| 17 | 336·247 | 2·985 | 2·503 | 9·59 | 45·57 | ·318 | 233·27 | 3·43 | ·40 |
| 22 | 336·219 | 3·001 | 2·517 | 8·94 | 46·22 | ·281 | 266·76 | 3·49 | ·35 |
| 27 | 336·179 | 3·014 | 2·531 | 8·21 | 46·83 | ·240 | 300·31 | 3·55 | ·29 |
| Sept. 1 | 336·129 | 3·023 | 2·545 | 7·41 | 47·41 | ·198 | 333·91 | 3·60 | ·24 |
| 6 | 336·068 | +3·028 | +2·559 | −6·53 | 47·93 | ·155 | 7·55 | 3·64 | +0·19 |
| | | | | | | | | 4353·68 | |

| Greenw. Noon. | Angle of Position of ♃'s Axis. | Latitude of Earth above ♃'s Equator. | Latitude of Sun above ♃'s Equator. | Annual Parallax. | Equat. Diam. | Greatest Phase. | Longitude of ♃'s Meridian directed to the Earth. | Diff. | Corr. of Long. |
|---|---|---|---|---|---|---|---|---|---|
| 1880. | ° | ° | ° | ° | ″ | ″ | ° | ° | ° |
| Sept. 11 | 335·999 | +3·030 | +2·573 | −5·59 | 48·39 | 0·115 | 41·23 | | +0·14 |
| | | | | | | | | 4353·71 | |
| 16 | 335·924 | 3·027 | 2·586 | 4·59 | 48·79 | ·078 | 74·94 | | ·09 |
| | | | | | | | | 3·72 | |
| 21 | 335·845 | 3·020 | 2·599 | 3·54 | 49·10 | ·047 | 108·66 | | ·05 |
| | | | | | | | | 3·72 | |
| 26 | 335·762 | 3·008 | 2·612 | 2·45 | 49·34 | ·023 | 142·38 | | ·03 |
| | | | | | | | | 3·71 | |
| Oct. 1 | 335·679 | 2·993 | 2·625 | 1·33 | 49·48 | ·007 | 176·09 | | +·01 |
| | | | | | | | | 3·69 | |
| 6 | 335·596 | 2·974 | 2·637 | −0·21 | 49·52 | ·000 | 209·78 | | ·00 |
| | | | | | | | | 3·66 | |
| 11 | 335·516 | 2·951 | 2·650 | +0·93 | 49·47 | ·003 | 243·44 | | ·00 |
| | | | | | | | | 3·60 | |
| 16 | 335·441 | 2·925 | 2·662 | 2·05 | 49·33 | ·016 | 277·04 | | −0·02 |
| | | | | | | | | 3·54 | |
| 21 | 335·371 | 2·897 | 2·674 | 3·15 | 49·09 | ·037 | 310·58 | | ·04 |
| | | | | | | | | 3·47 | |
| 26 | 335·307 | 2·866 | 2·686 | 4·21 | 48·77 | ·066 | 344·05 | | ·08 |
| | | | | | | | | 3·39 | |
| 31 | 335·251 | 2·834 | 2·698 | 5·22 | 48·37 | ·100 | 17·44 | | ·12 |
| | | | | | | | | 3·30 | |
| Nov. 5 | 335·203 | 2·802 | 2·710 | 6·18 | 47·89 | ·139 | 50·74 | | ·17 |
| | | | | | | | | 3·19 | |
| 10 | 335·162 | 2·769 | 2·721 | 7·07 | 47·35 | ·180 | 83·93 | | ·22 |
| | | | | | | | | 3·10 | |
| 15 | 335·129 | 2·737 | 2·732 | 7·88 | 46·76 | ·221 | 117·03 | | ·27 |
| | | | | | | | | 2·99 | |
| 20 | 335·104 | 2·705 | 2·743 | 8·62 | 46·12 | ·260 | 150·02 | | ·32 |
| | | | | | | | | 2·88 | |
| 25 | 335·087 | 2·675 | 2·754 | 9·28 | 45·45 | ·297 | 182·90 | | ·37 |
| | | | | | | | | 2·77 | |
| 30 | 335·077 | 2·647 | 2·765 | 9·85 | 44·76 | ·330 | 215·67 | | ·42 |
| | | | | | | | | 2 67 | |
| Dec. 5 | 335·074 | +2·621 | +2·776 | +10·33 | 44·05 | ·357 | 248·34 | | −0·46 |
| | | | | | | | | 4352·57 | |

| Greenw. Noon. | Angle of Position of ♃'s Axis. | Latitude of Earth above ♃'s Equator. | Latitude of Sun above ♃'s Equator. | Annual Parallax. | Equat. Diam. | Greatest Phase. | Longitude of ♃'s Meridian directed to the Earth. | Diff. | Corr. of Long. |
|---|---|---|---|---|---|---|---|---|---|
| 1830. | ° | ° | ° | ° | ″ | ″ | ° | ° | ° |
| Dec. 10 | 335·079 | +2·598 | +2·786 | +10·72 | 43·33 | 0·378 | 280·91 | | −0·50 |
| | | | | | | | | 4352·47 | |
| 15 | 335·092 | 2·577 | 2·796 | 11·02 | 42·61 | ·393 | 313·38 | | ·53 |
| | | | | | | | | 2·37 | |
| 20 | 335·112 | 2·559 | 2·806 | 11·25 | 41·91 | ·403 | 345·75 | | ·55 |
| | | | | | | | | 2·29 | |
| 25 | 335·139 | 2·544 | 2·816 | 11·39 | 41·21 | ·406 | 18·04 | | ·56 |
| | | | | | | | | 2·20 | |
| 30 | 335·175 | 2·532 | 2·826 | 11·46 | 40·53 | ·404 | 50·24 | | ·57 |
| | | | | | | | | 2·13 | |
| 1831. | | | | | | | | | |
| Jan. 4 | 335·220 | 2·523 | 2·836 | 11·44 | 39·87 | 0·396 | 82·37 | | 0·57 |
| | | | | | | | | 2·06 | |
| 9 | 335·273 | 2·516 | 2·845 | 11·36 | 39·23 | ·384 | 114·43 | | ·56 |
| | | | | | | | | 1·99 | |
| 14 | 335·335 | 2·513 | 2·854 | 11·21 | 38·62 | ·368 | 146·42 | | ·55 |
| | | | | | | | | 1·94 | |
| 19 | 335·406 | 2·512 | 2·863 | 10·99 | 38·04 | ·349 | 178·36 | | ·53 |
| | | | | | | | | 1·88 | |
| 24 | 335·487 | 2·514 | 2·872 | 10·71 | 37·49 | ·327 | 210·24 | | ·50 |
| | | | | | | | | 1·84 | |
| 29 | 335·579 | 2·518 | 2·880 | 10·37 | 36·97 | ·302 | 242·08 | | ·47 |
| | | | | | | | | 1·80 | |
| Feb. 3 | 335·681 | 2·524 | 2·889 | 9·99 | 36·48 | ·277 | 272·88 | | ·43 |
| | | | | | | | | 1·76 | |
| 8 | 335·795 | 2·533 | 2·897 | 9·56 | 36·02 | ·250 | 305·64 | | ·40 |
| | | | | | | | | 1·74 | |
| 13 | 335·920 | 2·543 | 2·905 | 9·08 | 35·59 | ·223 | 337·38 | | ·36 |
| | | | | | | | | 1·71 | |
| 18 | 336·056 | 2·556 | 2·913 | 8·56 | 35·19 | ·196 | 9·09 | | ·32 |
| | | | | | | | | 1·70 | |
| 23 | 336·204 | 2·570 | 2·921 | 8·01 | 34·83 | ·170 | 40·79 | | ·28 |
| | | | | | | | | 4351·68 | |
| 28 | 336·365 | +2·584 | +2·928 | +7·42 | 34·50 | 0·144 | 72·47 | | −0·24 |

The "annual parallax" is the difference of the Jovicentric longitudes of the Sun and the Earth, reckoned in the plane of *Jupiter's* equator. If the correction given in the last column is applied to the "longitude of ♃'s meridian directed to the Earth," the longitude of the meridian is found which bisects the illuminated disk. The First (or Zero) Meridian, from which the longitudes are reckoned is that which at the Greenwich midnight preceding January 1, 1872, was apparently directed to the Earth, and which is assumed to rotate at the daily rate of 870°·60, the corresponding period of rotation being $9^h\ 55^m\ 27^s{\cdot}08$. If suitable observations for ascertaining the true rate of rotation for different spots have been made during the apparitions of *Jupiter* from 1872 to 1878, for which Ephemerides were published, they have not yet come to my knowledge. The numerous observations of passages across the central meridian which have been made during the last season appear to have been confined to those of the red spot in the planet's southern hemisphere. Though their proper discussion must be deferred till the end of this exceptional appearance, and till all the accounts are available, a preliminary examination of the results will not be uninstructive, and I give, therefore, a list of the observed passages which have hitherto come to my knowledge, together with the corresponding longitudes of the meridian which bisected the illuminated disk, as derived from the Ephemeris, and also with the longitudes corrected by the quantity $-0°{\cdot}17$ ($t$ – Oct. 17·0 1879). The period of rotation corresponding to the daily rate 870°·43 is $9^h\ 55^m\ 34^s{\cdot}1$. A small correction, depending on the Jovicentric latitude of the spot and on the correction in the last column of the Ephemeris, is still required in those cases in which the observed passage does not refer to the meridian which is equidistant from the two limbs (one of which is affected by the phase), but to the line perpendicular upon and bisecting the apparent equatoreal diameter. Except at opposition, this line does not pass through the poles of the planet, and is not a meridian line, and though the difference is small for spots not far from the equator, it may become very sensible for spots in higher latitudes.

The sources from which the list is derived are mentioned *post*, p. 429 : the times are Ath. Athens, Br. Brussels, Gr. Greenwich, Mo. Moscow mean times, as supplied by the observers ; in the cases, however, where seconds are given, tenths of minutes have been substituted for the sake of uniformity.

| 1879. | Observed Passages. Prec. End. h m | Middle. h m | Foll. End. h m | Observer. | | Corresponding Longitudes. Prec. End. | Rot. 870°·60. Middle. | Foll. End. | Prec. End. | Rot. 870°·43. Middle. | Foll. End. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | ° | ° | ° | ° | ° | ° |
| July 10 | — | 19 11·3 | — | Gr. | Pritchett | — | 243·9 | — | — | 260·6 | — |
| 22 | — | 19 1·3 | — | „ | „ | — | 246·2 | — | — | 260·9 | — |
| 26 | — | 12 25·0 | — | „ | Pratt | — | 249·4 | — | — | 263·4 | — |
| 28 | 14 18·4 | 14 38·4 | 14 48·5 | Br. | Niesten | 248·8 | 260·9 | 267·1 | 262·5 | 274·6 | 280·7 |
| 29 | 10 4·7 | 10 19·8 | 10 34·8 | „ | „ | 246·2 | 255·3 | 264·4 | 259·8 | 268·8 | 277· |
| 31 | 11 30·6 | 11 54·6 | 12 15·6 | „ | „ | 239·5 | 254·0 | 266·7 | 252·7 | 267·2 | 279· |
| Aug. 2 | 12 50 | 13 27·8 | — | „ | „ | 229·0 | 249·4 | — | 241·8 | 262·3 | — |
| 4 | — | 15 8·2 | 15 29·8 | „ | „ | — | 254·0 | 267·1 | — | 266·5 | 279·6 |
| 6 | — | 16 25·3 | — | Gr. | Pritchett | — | 252·7 | — | — | 264·8 | — |
| 7 | 11 30 | — | — | „ | Brewin | 224·9 | — | — | 236·8 | — | — |
| 7 | — | 22 12·3 | — | „ | Pritchett | — | 253·2 | — | — | 265·2 | — |
| 8 | — | 18 2·3 | — | „ | „ | — | 252·8 | — | — | 264·6 | — |
| 9 | 13 38·5 | 14 0·5 | — | Br. | Niesten | 233·5 | 246·8 | — | 245·1 | 258·4 | — |
| 10 | 9 39·1 | 10 8·1 | 10 31·0 | „ | „ | 239·4 | 256·9 | 270·8 | 250·9 | 268·4 | 281·5 |
| 12 | 11 17 | 11 38 | 12 1·5 | „ | „ | 240·1 | 252·8 | 267·0 | 251·2 | 263·9 | 278·1 |
| 15 | — | 18 42·3 | — | Gr. | Pritchett | — | 252·1 | — | — | 263·7 | — |
| 16 | — | 14 34·1 | — | „ | „ | — | 252·8 | — | — | 263·2 | — |
| 17 | 10 28·6 | 10 43·9 | 11 7·2 | Br. | Niesten | 244·5 | 253·8 | 267·8 | 254·8 | 264·1 | 278·1 |
| 18 | — | 16 11·9 | — | Gr. | Pritchett | — | 253·4 | — | — | 263·5 | — |
| 20 | — | 17 48·3 | — | „ | „ | — | 253·2 | — | — | 262·9 | — |

| 1879. | Observed Passages. Prec. End. h m | Middle. h m | Foll. End. h m | Observer. | | Corresponding Longitudes. Rot. 870°·60. Prec. End. ° | Middle. ° | Foll. End. ° | Rot. 870°·43. Prec. End. ° | Middle. ° | Foll. End. ° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Aug. 22 | — | 19 25·3 | — | Gr. | Pritchett | — | 253·3 | — | — | 262·7 | — |
| 24 | — | 21 0·6 | — | ,, | ,, | — | 252·4 | — | — | 261·4 | — |
| 25 | — | 16 51·8 | — | ,, | ,, | — | 252·7 | — | — | 261·6 | — |
| 27 | — | 18 29·3 | — | ,, | ,, | — | 253·1 | — | — | 261·7 | — |
| 28 | — | 14 19·0 | — | ,, | ,, | — | 252·5 | — | — | 260·9 | — |
| 29 | — | 10 14 | — | ,, | Backhouse | — | 255·1 | — | — | 263·3 | — |
| 29 | — | 20 7·3 | — | ,, | Pritchett | — | 253·8 | — | — | 262·0 | — |
| 30 | — | 15 58·1 | — | ,, | ,, | — | 253·9 | — | — | 261·9 | — |
| 31 | 11 33·6 | 12 4·4 | — | Br. | Niesten | 234·1 | 253·7 | — | 242·0 | 261·6 | — |
| Sept. 1 | — | 17 33·9 | — | Gr. | Pritchett | — | 253·3 | — | — | 261·0 | — |
| 3 | 9 10 | — | — | Br. | Niesten | 239·5 | — | — | 246·9 | — | — |
| 3 | — | 9 19 | — | Gr. | Pratt | — | 255·5 | — | — | 262·9 | — |
| 3 | — | 19 9·8 | — | ,, | Pritchett | — | 252·7 | — | — | 260·1 | — |
| 4 | — | 15 2·3 | — | ,, | ,, | — | 253·8 | — | — | 261·0 | — |
| 6 | — | 16 40·7 | — | ,, | ,, | — | 254·7 | — | — | 261·6 | — |
| 8 | — | 8 25 | 8 55 | ,, | MacCance | — | 256·4 | 274·6 | — | 263·0 | 281·1 |
| 8 | — | 18 15·8 | — | ,, | Pritchett | — | 253·7 | — | — | 260·2 | — |
| 10 | — | 10 1·5 | — | ,, | Pratt | — | 256·2 | — | — | 262·4 | — |
| 10 | — | 19 51·7 | — | ,, | Pritchett | — | 253·1 | — | — | 259·2 | — |
| 11 | — | 8 25·9 | — | Mo. | Bredichin | — | 258·2 | — | — | 264·3 | — |

| 1879. | Observed Passages. Prec. End. h m | Middle. h m | Foll. End. h m | Observer. | | Corresponding Longitudes. Prec. End. ° | Rot. 870°·60. Middle. ° | Foll. End. ° | Prec. End. ° | Rot. 870°·43. Middle. ° | Foll. End ° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Sept. 12 | — | 11 59·1 | — | Br. | Niesten | — | 258·1 | — | — | 264·0 | — |
| 13 | — | 17 22·3 | — | Gr. | Pritchett | — | 254·8 | — | — | 260·5 | — |
| 14 | — | 13 13·9 | — | ,, | ,, | — | 255·3 | — | — | 260·9 | — |
| 15 | 8 35 | 9 7 | — | ,, | MacCance | 237·4 | 256·7 | — | 242·8 | 262·1 | — |
| 15 | — | 11 43·7 | — | Mo. | Bredichin | — | 260·6 | — | — | 266·0 | — |
| 15 | — | 18 59·7 | — | Gr. | Pritchett | — | 255·1 | — | — | 260·4 | — |
| 16 | — | 14 50·9 | — | ,, | ,, | — | 255·4 | — | — | 260·5 | — |
| 17 | — | 13 18·6 | — | Mo. | Bredichin | — | 259·4 | — | — | 264·4 | — |
| 18 | — | 9 11·3 | — | ,, | ,, | — | 260·5 | — | — | 265·4 | — |
| 20 | — | 10 48·2 | — | ,, | ,, | — | 260·5 | — | — | 265·0 | — |
| 20 | — | 18 6·3 | — | Gr. | Pritchett | — | 256·2 | — | — | 260·7 | — |
| 22 | 9 37·1 | 10 0·7 | 10 30·1 | Br. | Niesten | 239·1 | 253·4 | 271·1 | 243·3 | 257·6 | 275·3 |
| 22 | — | 12 25·1 | — | Mo. | Bredichin | — | 260·4 | — | — | 264·6 | — |
| 22 | 9 29 | 9 33 | 10 23 | Gr. | Backhouse | 244·8 | 260·5 | 277·4 | 249·0 | 264·7 | 281·6 |
| 23 | — | 8 17·8 | — | Mo. | Bredichin | — | 261·5 | — | — | 265·6 | — |
| 24 | — | 11 55 | — | Br. | Niesten | — | 263·8 | — | — | 267·6 | — |
| 25 | — | 9 55·7 | — | Mo. | Bredichin | — | 262·1 | — | — | 265·7 | — |
| 25 | — | 7 44·5 | — | Br. | Niesten | — | 263·0 | — | — | 266·7 | — |
| 25 | — | 17 12·3 | — | Gr. | Pritchett | — | 256·9 | — | — | 260·5 | — |
| 26 | — | 13 3·1 | — | ,, | ,, | — | 256·9 | — | — | 260·4 | — |

| 1879. | Observed Passages. Prec. End. h m | Middle. h m | Foll. End. h m | Observer. | Corresponding Longitudes. Rot. 870°·60. Prec. End. ° | Middle. ° | Foll. End. ° | Rot. 870°·43. Prec. End. ° | Middle. ° | Foll. End. ° |
|---|---|---|---|---|---|---|---|---|---|---|
| Sept. 27 | 8 44·6 | 9 11·9 | 9 41·9 | Br. Niesten | 240·6 | 257·2 | 275·3 | 244·0 | 260·5 | 278·6 |
| 27 | 9 19·8 | 9 48·3 | 10 23·3 | Berlin Lohse | 240·1 | 257·3 | 278·5 | 243·5 | 260·7 | 281·8 |
| 27 | — | 8 59 | — | Gr. Pratt | — | 259·9 | — | — | 263·3 | — |
| 27 | — | 11 35·6 | — | Mo. Bredichin | — | 263·8 | — | — | 267·1 | — |
| 27 | — | 18 51·3 | — | Gr. Pritchett | — | 258·1 | — | — | 261·3 | — |
| 29 | — | 10 43 | — | ,, Backhouse | — | 264·1 | — | — | 267·1 | — |
| 30 | — | 16 19·6 | — | ,, Pritchett | — | 258·3 | — | — | 261·0 | — |
| Oct. 1 | — | 12 15·5 | — | ,, Pratt | — | 261·3 | — | — | 264·0 | — |
| 2 | 7 59·4 | 8 19·4 | 8 44·8 | Br. Niesten | 246·6 | 258·6 | 274·0 | 249·1 | 261·1 | 276·5 |
| 2 | — | 10 43·1 | — | Mo. Bredichin | — | 265·2 | — | — | 267·7 | — |
| 3 | — | 6 30·9 | — | ,, ,, | — | 263·4 | — | — | 265·7 | — |
| 5 | — | 8 10·8 | — | ,, ,, | — | 265·0 | — | — | 267·0 | — |
| 5 | — | 15 26·1 | — | Gr. Pritchett | — | 259·0 | — | — | 261·0 | — |
| 6 | — | 11 22·3 | — | ,, Pratt | — | 262·2 | — | — | 264·0 | — |
| 6 | 10 59 | 11 23 | — | ,, Backhouse | 248·2 | 262·7 | — | 250·0 | 264·5 | — |
| 6 | 11 25·7 | 11 42 | 12 3 | Br. Niesten | 253·7 | 263·6 | 276·3 | 255·5 | 265·4 | 278·1 |
| 7 | 7 12·5 | 7 32·5 | 7 53·5 | ,, ,, | 251·3 | 263·4 | 273·1 | 252·9 | 265·0 | 274·7 |
| 7 | — | 17 4·3 | — | Gr. Pritchett | — | 259·6 | — | — | 261·2 | — |
| 8 | — | 12 56·7 | — | ,, ,, | — | 260·5 | — | — | 262·0 | — |
| 11 | — | 13 7·5 | — | Mo. Bredichin | — | 268·0 | — | — | 269·0 | — |

| 1879. | Observed Passages. Prec. End. h m | Middle. h m | Foll. End. h m | Observer. | | Corresponding Longitudes. Prec. End. | Rot. 870°·60. Middle. | Foll. End. | Prec. End. | Rot. 870°·43. Middle. | Foll. End. |
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| | | | | | | ° | ° | ° | ° | ° | ° |
| Oct. 12 | — | 16 10·2 | — | Gr. | Pritchett | — | 259·9 | — | — | 260·7 | — |
| 13 | — | 12 1·0 | — | ,, | ,, | — | 259·8 | — | — | 260·4 | — |
| 14 | — | 10 32·2 | — | Mo. | Bredichin | — | 265·9 | — | — | 266·3 | — |
| 14 | — | 17 50·5 | — | Gr. | Pritchett | — | 261·6 | — | — | 262·0 | — |
| 15 | — | 13 41·1 | — | ,, | ,, | — | 261·5 | — | — | 261·8 | — |
| 16 | 9 32·3 | 9 51·3 | 10 12·3 | Br. | Niesten | 251·1 | 262·6 | 275·3 | 251·2 | 262·7 | 275·4 |
| 17 | — | 8 1·8 | — | Mo. | Bredichin | — | 266·7 | — | — | 266·6 | — |
| 17 | — | 15 18·9 | — | Gr. | Pritchett | — | 261·8 | — | — | 261·7 | — |
| 18 | — | 11 11·7 | — | ,, | ,, | — | 262·9 | — | — | 262·6 | — |
| 19 | — | 17 0·3 | — | ,, | ,, | — | 264·2 | — | — | 263·7 | — |
| 20 | - | 5 32·5 | — | Mo. | Bredichin | — | 268·0 | — | — | 267·5 | .. |
| 20 | — | 12 50·4 | — | Gr. | Pritchett | — | 263·7 | — | — | 263·1 | — |
| 22 | — | 14 29·4 | — | ,, | ,, | — | 264·6 | — | — | 263·7 | — |
| 24 | — | 8 50·3 | — | Mo. | Bredichin | — | 269·9 | — | — | 268·6 | — |
| 24 | — | 16 7·4 | — | Gr. | Pritchett | — | 265·0 | — | — | 263·7 | — |
| 25 | — | 11 59·6 | — | ,, | ,, | — | 265·7 | — | — | 264·3 | — |
| 26 | — | 7 50 | — | ,, | Pratt | — | 265·3 | — | — | 263·7 | — |
| 27 | — | 13 38·0 | — | ,, | Pritchett | — | 266·2 | — | — | 264·4 | — |
| 28 | — | — | 9 56 | ,, | Backhouse | — | — | 282·6 | — | — | 280·6 |
| 29 | — | 15 15·3 | — | ,, | Pritchett | — | 266·1 | — | — | 264·0 | — |

| 1879. | Observed Passages. Prec. End. h m | Middle. h m | Foll. End. h m | Observer. | | Corresponding Longitudes. Rot. 870°·60. Prec. End. ° | Middle. ° | Foll. End. ° | Rot. 870°·43. Prec. End. ° | Middle. ° | Foll. End. ° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Oct. 30 | — | 11 6·3 | — | Gr. | Pritchett | — | 266·1 | — | — | 263·8 | — |
| Nov. 1 | — | 5 24·6 | — | Mo. | Bredichin | — | 269·7 | — | — | 267·1 | — |
| 2 | — | 8 35·3 | — | Gr. | Pratt | — | 266·4 | — | — | 263·6 | — |
| 2 | — | 8 45 | — | „ | Green | — | 272·2 | — | — | 269·4 | — |
| 3 | — | 14 20·8 | — | „ | Pritchett | — | 265·7 | — | — | 262·9 | — |
| 5 | — | 6 4·3 | — | „ | Pratt | — | 266·6 | — | — | 263·3 | — |
| 5 | — | 16 0·7 | — | „ | Pritchett | — | 267·1 | — | — | 263·7 | — |
| 6 | — | 11 53·0 | — | „ | „ | — | 267·8 | — | — | 264·4 | — |
| 7 | 7 13 | 7 44 | 8 13 | „ | Gledhill | 249·1 | 267·8 | 285·3 | 245·5 | 264·2 | 281·7 |
| 7 | 7 15 | 7 47 | — | „ | Green | 250·3 | 269·6 | — | 246·7 | 266·0 | — |
| 7 | — | — | 8 17 | „ | Backhouse | — | — | 287·8 | — | — | 284·1 |
| 10 | 6 17·9 | 6 50·5 | 7 9·6 | Ath. | Schmidt | 249·8 | 269·5 | 281·1 | 245·7 | 265·4 | 277·0 |
| 10 | — | — | 5 44 | Gr. | Backhouse | — | — | 286·7 | — | — | 282·6 |
| 10 | — | 15 7·2 | — | „ | Pritchett | — | 267·2 | — | — | 263·0 | — |
| 12 | — | 6 45 | — | „ | Green | — | 264·6 | — | — | 260·1 | — |
| 12 | 7 17·7 | 8 24·7 | 8 47·6 | Ath. | Schmidt | 226·9 | 267·4 | 281·2 | 222·5 | 263·0 | 276·8 |
| 12 | 6 20 | 6 50 | 7 20 | Gr. | Gledhill | 249·4 | 267·6 | 285·7 | 245·0 | 263·1 | 281·2 |
| 12 | — | 6 50 | — | „ | Pratt | — | 267·6 | — | — | 263·1 | — |
| 12 | — | 9 25·8 | — | Mo. | Bredichin | — | 270·9 | — | — | 266·4 | — |
| 14 | 7 59 | 8 25 | 9 6 | Gr. | Gledhill | 250·2 | 265·9 | 290·7 | 245·4 | 261·1 | 285·9 |

| 1879. | Observed Passages. Prec. End. h m | Middle. h m | Foll. End. h m | Observer. | Corresponding Longitudes. Prec. End. | Rot. 870°·60. Middle. | Foll. End. | Prec. End. | Rot. 870°·43. Middle. | Foll. End. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | ° | ° | ° | ° | ° | ° |
| Nov. 15 | — | 14 13·3 | — | Gr. Pritchett | — | 267·0 | — | — | 261·9 | — |
| 17 | 5 35 | — | — | „ Backhouse | 254·6 | — | — | 249·3 | — | — |
| 19 | — | 7 35 | — | „ Green | — | 268·0 | — | — | 262·3 | — |
| 19 | — | 7 35·5 | — | „ Pratt | — | 268·3 | — | — | 262·6 | — |
| 22 | — | 14 58·3 | — | „ Pritchett | — | 267·3 | — | — | 261·1 | — |
| 23 | — | 10 52·3 | — | „ „ | — | 269·0 | — | — | 262·7 | — |
| 24 | 7 52·7 | 8 23·0 | 8 50·5 | Ath. Schmidt | 253·5 | 271·8 | 288·5 | 247·0 | 265·3 | 282·0 |
| 25 | — | 5 7·9 | — | Mo. Bredichin | — | 270·9 | — | — | 264·2 | — |
| 25 | — | 12 32·8 | — | Gr. Pritchett | — | 270·6 | — | — | 263·9 | — |
| 26 | 8 18·9 | 8 43·9 | 9 8·9 | Br. Niesten | 257·1 | 272·2 | 287·3 | 250·2 | 265·3 | 280·4 |
| 28 | — | 10 5·3 | — | Gr. Pritchett | — | 272·7 | — | — | 265·5 | — |
| 29 | 5 39·3 | 6 4·3 | 6 34·3 | Br. Niesten | 251·8 | 266·9 | 285·1 | 244·5 | 259·6 | 277·7 |
| 29 | 5 15 | 5 50 | 6 30 | Gr. Gledhill | 247·7 | 268·9 | 293·0 | 240·4 | 261·5 | 285·7 |
| 29 | — | 5 53 | 6 21 | „ Backhouse | — | 270·7 | 287·6 | — | 263·3 | 280·3 |
| 29 | — | 15 49·3 | — | „ Pritchett | — | 271·1 | — | — | 263·7 | — |
| 30 | — | 11 40·3 | — | „ „ | — | 271·0 | — | — | 263·5 | — |
| Dec. 1 | 7 3 | 7 35 | — | „ Gledhill | 253·9 | 273·2 | — | 246·2 | 265·5 | — |
| 2 | — | 3 53·4 | — | Br. Niesten | — | 279·1 | — | — | 271·2 | — |
| 3 | 8 45 | 9 10 | — | Gr. Gledhill | 256·3 | 271·4 | — | 248·3 | 263·4 | — |
| 4 | — | 7 33·2 | — | Mo. Bredichin | — | 272·5 | — | — | 264·3 | — |

| 1879. | Observed Passages. Prec. End. h m | Middle. h m | Foll. End. h m | Observer. | Corresponding Longitudes. Rot. 870°·60. Prec. End. ° | Middle. ° | Foll. End. ° | Rot. 870°·43. Prec. End. ° | Middle. ° | Foll. End. ° |
|---|---|---|---|---|---|---|---|---|---|---|
| Dec. 4 | 6 9·3 | 6 50·6 | 7 0·7 | Ath. Schmidt | 255·2 | 280·2 | 286·3 | 247·0 | 272·0 | 278·1 |
| 6 | — | 6 36 | — | Gr. Pratt | — | 269·6 | — | — | 261·0 | — |
| 6 | 6 28·9 | 6 54·6 | 7 23·9 | Br. Niesten | 254·7 | 270·3 | 288·0 | 246·2 | 261·8 | 279·4 |
| 6 | 6 14 | 6 40 | 7 10 | Gr. Backhouse | 256·3 | 272·0 | 290·1 | 247·7 | 263·4 | 281·6 |
| 6 | 6 5 | 6 45 | 7 15 | ,, Gledhill | 250·8 | 275·0 | 293·1 | 242·3 | 266·5 | 284·6 |
| 6 | — | 6 45 | 7 15 | ,, Knott | — | 275·0 | 293·1 | — | 266·5 | 284·6 |
| 6 | 7 58·9 | 8 18·8 | 8 52·3 | Ath. Schmidt | 262·3 | 274·3 | 294·6 | 253·8 | 265·8 | 286·1 |
| 8 | 8 2·2 | 8 29·2 | 8 55·2 | Br. Niesten | 251·9 | 268·2 | 283·9 | 243·0 | 259·3 | 275·0 |
| 8 | 7 41 | 8 21 | 8 46 | Gr. Gledhill | 249·7 | 273·8 | 288·9 | 240·7 | 264·9 | 280·0 |
| 9 | 4 2·0 | 4 22·3 | 4 56·8 | Br. Niesten | 257·1 | 269·4 | 290·2 | 248·1 | 260·3 | 281·2 |
| 11 | 5 22 | — | — | Gr. MacCance | 256·8 | — | — | 247·4 | — | — |
| 11 | | 15 40·6 | — | ,, Pritchett | — | 270·7 | — | — | 261·3 | — |
| 12 | — | 11 33·0 | — | ,, ,, | — | 271·5 | — | — | 261·9 | — |
| 13 | — | 7 35 | — | ,, Backhouse | — | 278·0 | — | — | 268·3 | — |
| 16 | 4 49·9 | 5 16·9 | 5 44·4 | Br. Niesten | 258·8 | 275·1 | 291·7 | 248·6 | 264·9 | 281·5 |
| 16 | — | 14 50·3 | — | Gr. Pritchett | — | 272·3 | — | — | 262·0 | — |
| 18 | 6 18·3 | 6 48·3 | 7 15·3 | Br. Niesten | 253·0 | 271·1 | 287·5 | 242·4 | 260·6 | 276·9 |
| 20 | — | 8 30 | — | ,, ,, | — | 273·4 | — | — | 262·4 | — |
| 22 | — | 9 52·6 | — | Gr. Pritchett | — | 274·6 | — | — | 263·3 | — |
| 23 | 5 38·4 | 6 2·9 | 6 32·9 | Br. Niesten | 260·8 | 275·6 | 293·7 | 249·3 | 264·1 | 282·3 |
| 23 | 5 18 | 5 48 | 6 15 | Gr. Backhouse | 259·0 | 277·2 | 293·5 | 247·6 | 265·7 | 282·0 |

| 1879. | Observed Passages. Prec. End. h m | Middle. h m | Foll. End. h m | Observer. | | Corresponding Longitudes. Prec. End. ° | Rot. 870°·60. Middle. ° | Foll. End. ° | Rot. 870°·43. Prec. End. ° | Middle. ° | Foll. End. ° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Dec. 24 | — | 11 31·8 | — | Gr. | Pritchett | — | 275·3 | | — | 263·7 | — |
| 25 | 7 1 | 7 27 | 7 56 | „ | Backhouse | 262·0 | 277·7 | 295·3 | 250·2 | 266·0 | 283·5 |
| 26 | 4 35·2 | 4 51·5 | 5 14·1 | Ath. | Schmidt | 266·9 | 276·7 | 290·4 | 255·0 | 264·8 | 278·5 |
| 26 | — | 13 10·3 | — | Gr. | Pritchett | — | 275·6 | — | — | 263·6 | — |
| 1880. | | | | | | | | | | | |
| Jan. 4 | 5 15 | 5 42 | 6 11 | „ | MacCance | 261·6 | 277·9 | 295·4 | 248·1 | 264·4 | 282·0 |
| 12 | — | 12 16·3 | — | „ | Pritchett | — | 279·1 | — | — | 264·2 | — |
| 14 | — | 13 54·3 | — | „ | „ | — | 279·0 | — | — | 263·8 | — |
| 17 | — | 4 5·0 | — | Mo. | Bredichin | — | 283·0 | — | — | 267·4 | — |
| 17 | — | 11 23·4 | — | Gr. | Pritchett | — | 278·8 | — | — | 263·1 | — |
| 19 | — | 5 43·4 | — | Mo. | Bredichin | — | 283·2 | — | — | 267·2 | — |
| 19 | — | 13 0·8 | — | Gr. | Pritchett | — | 278·4 | — | — | 262·3 | — |
| 26 | 3 47·9 | 4 12·9 | 4 42·9 | Br. | Niesten | 266·1 | 281·2 | 299·3 | 248·9 | 264·0 | 282·1 |
| 26 | — | — | 4 30 | Gr. | Gledhill | — | — | 302·1 | — | — | 284·9 |
| 28 | 5 15·3 | 5 43·3 | 6 11 | Br. | Niesten | 259·6 | 276·5 | 293·2 | 242·0 | 258·9 | 275 7 |
| 28 | 5 10 | 5 40 | 6 15 | Gr. | Gledhill | 266·9 | 285·1 | 306·0 | 249·4 | 267·5 | 288·5 |
| Feb. 4 | 5 30 | — | — | „ | Brewin | 251·4 | — | — | 232·7 | — | — |
| 4 | 6 4 | 6 30 | 6 58 | Br. | Niesten | 261·4 | 277·1 | 294·0 | 242·6 | 258·4 | 275·3 |
| 5 | — | 12 6·3 | — | Gr. | Pritchett | — | 281·3 | — | — | 262·3 | — |
| 7 | — | 13 43·3 | — | „ | „ | — | 280·6 | — | — | 261·4 | — |
| 22 | 6 6 | — | — | Cambr. | Trouvelot | 271·3 | — | — | 249·4 | — | — |

The times of the observed passages contained in the list are taken from the following sources:—

| | |
|---|---|
| T. W. Backhouse, Sunderland. | *Monthly Notices*, vol. xl. p. 157. |
| Th. Bredichin, Moscow. | *Annales de l'Observatoire de Moscou*, tom. vi. 2 livraison, p. 105, 106. |
| T. D. Brewin, Leicester. | *Monthly Notices*, vol. xl. p. 377. |
| J. Gledhill, Halifax. | *The Observatory*, vol. iii. p. 280 and 355. |
| N. Green, St. John's Wood. | Private communication. |
| G. Knott, Cuckfield. | *Astron. Register*, vol. xviii. p. 91. |
| O. Lohse, Potsdam. | *Astron. Nachr.* No. 2282. |
| J. L. Mac Cance, Putney. | Private communication. |
| L. Niesten, Brussels. | *Bulletins de l'Acad. Roy. de Belgique*, t. xlviii. no. 12. |
| | [The observations from November 26 to February 4 have been kindly communicated by letter.] |
| H. Pratt, Brighton. | *Monthly Notices*, vol. xl. p, 154. |
| C. H. Pritchett, Glasgow, Miss. | From a list kindly communicated in MSS., in which the observed times are already given in Greenwich M. T. |
| J. Schmidt, Athens. | *Astron. Nachr.* No. 2309. |
| L. Trouvelot, Cambridge, Mass. | *Observatory*, vol. iii. p. 417, where for G. M. T. must be read Cambr. M. T. |

An examination of the longitudes in the three last columns will give at least some preliminary indications of the errors of the observations and of the deviations of the motion of the red spot from regularity. The discrepancies between the observations of different observers and on different days point to the existence of some grave sources of error and show the necessity for greater care and caution. On the other hand, the consideration of the fair agreement frequently found may give some assurance that the observations are worth making, and observers may perhaps be induced thereby to spend sufficient time and patience in watching the passages across the central meridian of all well-marked points on the planet's surface, so that at last proper observations may be available for investigating the various motions which are going on there.

## *The possible Ten-Month Period of Variation in Latitude.*

By A. M. W. Downing, Esq.

If from any cause the axis of rotation of the Earth does not coincide with the axis of figure the former will revolve round the latter in a period which is equal to

$$\frac{A}{C-A'}$$

employing the notation usual in works on dynamics. The most probable value of this period appears to be 306 mean solar days. The apparent pole of the Earth would therefore travel round the pole of the adjacent principal axis in this time, and give rise to what may be called a ten-month period of variation in latitude as determined from meridian zenith distances of stars observed above and below the pole. Clerk Maxwell has examined (*Trans. Edin. R. S.* vol. xxi. p. 569) the observations of *Polaris* made at the Royal Observatory, Greenwich, during the years 1851–4 to ascertain whether any period of this kind could be detected in the resulting latitudes, the apparent co-latitude being found from the observations made during each month. There appeared to be a slight indication of a maximum in 1851 March, 1852 Feb., 1852 Dec., 1853 Nov., and 1854 Sept. The author considers, however, that "this result is to be regarded as very doubtful," and that "more observations would be required to establish the existence of so small a variation at all."

In the present paper I have discussed the Greenwich observation of *Polaris* made during the ten years 1868–1877, with the view of determining the amount of this variation (if it exists), and also the time of maximum effect.

The zenith distances of *Polaris*, as extracted from the volumes of *Greenwich Observations*, have been corrected, where necessary, for errors in the screws of the microscope-micrometers by applying the quantities given in the Introduction to the Nine-Year Catalogue; also for error in the Transit-Circle thermometer, a correction of $-0^{\circ}\cdot5$ being applied to all readings, and the corresponding correction applied to the zenith distance. The refractions used are those of Bessel's *Tabulæ Regiomontanæ*. The correction for discordance of direct and reflexion observations has been also applied.

The observations have then been grouped—usually by months, except in cases where there is an insufficient number of observations, either above or below the pole. The apparent co-latitude has then been formed from the observations of each group, the effect of uncorrected aberration and of annual parallax being practically eliminated by this method of treating the observations. The following table exhibits the series of co-latitudes

thus formed—the degrees and minutes being 38° 31′ in all cases :—

| | 1868. | 1869. | 1870. | 1871. | 1872. | 1873. | 1874. | 1875. | 1876. | 1877. |
|---|---|---|---|---|---|---|---|---|---|---|
| | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ |
| Jan. | 22·12 | 22·28 | — | — | 21·92 | 21·96 | — | — | — | 22·32 |
| Feb. | 21·77 | — | — | — | — | — | — | — | — | 21·62 |
| March | 21 99 | 22·14 | 22·25 | 22·61 | 21·66 | 21·52 | 21·65 | 22·20 | 21·88 | 21·97 |
| April | 21·45 | 21·98 | 21·40 | 23·29 | 21·65 | 21·50 | 21·21 | 21·43 | 21·25 | 21·65 |
| May | 22·10 | 21·56 | 21·24 | 22·14 | 21·83 | 21 92 | 21·69 | 21·72 | 21·39 | 21·86 |
| June | 21·84 | 21·07 | 21·81 | 21·53 | 21·46 | 21·37 | 21·12 | 21·41 | 21·51 | 21·39 |
| July | 21·55 | — | 21·38 | 21·63 | — | 21·84 | 20·71 | 21·76 | 21·28 | 20·52 |
| August | 21·73 | — | 21·98 | — | — | 21·62 | — | 21·37 | 21·29 | 21·70 |
| Sept. | 21·48 | — | 21·55 | 21·61 | 21·64 | 21·59 | 22·02 | 21·64 | 21·50 | 20·86 |
| Oct. | 22·19 | 21·65 | 21·16 | 21·39 | 21·55 | 21·57 | 21·92 | 22·28 | 22·08 | 21·77 |
| Nov. | 22·41 | 21·93 | 21·70 | 21·98 | 21·70 | 21·87 | 22·05 | 22·46 | 21·81 | 21·96 |
| Dec. | 21·89 | 22·51 | 22·96 | 21·88 | 22·15 | 22·36 | 21·82 | 22·89 | 22·50 | 21·52 |

There appears, therefore, to be an annual variation in the Greenwich apparent co-latitudes (the possibility of the existence of which was pointed out by me in *M. N.* vol. xl. p. 86), but there is no variation with a ten-month period of any considerable amount.

I proceed now to calculate the amount of the latter variation, assuming its period to be 306 days.

Let $r$ be the angular distance between the moveable and fixed poles, and $\theta$ the angle which the great circle joining them makes with the Greenwich meridian, referred to the fixed pole, at the time 1872·0. Also let $\phi$ be the apparent co-latitude, and $\phi_0$ the co-latitude referred to the fixed pole, then we have

$$\begin{aligned}\phi &= \phi_0 + r\cos\{\theta + 429{\cdot}4(t-1872)\}\\ &= \phi_0 + r\cos\theta\cos 429{\cdot}4(t-1872) - r\sin\theta\sin 429{\cdot}4(t-1872).\end{aligned}$$

Let

$$\begin{aligned}x &= r\cos\theta,\\ y &= r\sin\theta,\\ z &= \text{correction to the assumed value of } \phi_0,\ 38^\circ\ 31'\ 21''{\cdot}90,\\ n &= 38^\circ\ 31'\ 21''{\cdot}90 - \phi;\end{aligned}$$

then the equations of condition are of the form

$$ax + by + cz + n = 0.$$

If $m$ and $m'$ be the number of observations above and below the pole respectively in each group, the weight of the resulting

apparent co-latitude, and therefore of the corresponding equation of condition, is

$$\frac{4mm'}{m+m'}.$$

Equations of conditions have been formed in this manner for each of the determinations of co-latitude (100 in number) given in the table above; these have been solved by the method of least squares, and the following normal equations obtained:—

$$\begin{aligned} +629{\cdot}66x + 11{\cdot}78y + 10{\cdot}58z + 44{\cdot}08 &= 0, \\ +11{\cdot}78x + 544{\cdot}25y + 7{\cdot}88z + 18{\cdot}76 &= 0, \\ +10{\cdot}58x + 7{\cdot}88y + 1179{\cdot}z + 105{\cdot}86 &= 0. \end{aligned}$$

From these I find

$$\begin{aligned} x &= -0''{\cdot}068 \pm 0''{\cdot}015, \\ y &= -0{\cdot}032 \pm 0{\cdot}016, \\ z &= -0{\cdot}089 \pm 0{\cdot}011. \end{aligned}$$

the probable error of a single determination of co-latitude of weight unity being

$$\pm 0''{\cdot}384.$$

From the above values of $x$ and $y$ we have

$$\begin{aligned} r &= 0''{\cdot}075 \pm 0''{\cdot}015, \\ \theta &= 205^{\circ}{\cdot}0 \pm 6^{\circ}{\cdot}7. \end{aligned}$$

Although the value thus found for $r$ is so small, still it will be remarked that it is five times greater than its probable error; so that, supposing we may assume that the variation in question remains constant for ten years, its reality may be asserted with some degree of confidence. But it is evidently much too small to be detected by mere inspection of the observations, which appears to have been the only process resorted to by Clerk Maxwell in the paper referred to above.

The value of $\theta$ which I have found indicates that the apparent latitude of Greenwich was a maximum on 1872, Oct. 12. Now, Prof. C. A. F. Peters, in his well-known work *Recherches sur la parallaxe des étoiles fixes*, has investigated this question of variability of latitude, making use of the Pulkowa observavations of *Polaris* from 1842 March to 1843 April. He found $r = 0''{\cdot}079 \pm 0''{\cdot}017$, and for the time when the latitude of Pulkowa was a maximum 1842 Nov. 16. If the variation remain constant the latitude should, therefore, again have been a maximum on 1872 Nov. 16; and the longitude of Pulkowa being $2^h$ east, the latitude of Greenwich would be maximum $\frac{1}{12}$ of the

period, or about 26 days earlier; that is, about 1872, Oct. 21. Prof. Peters' results, therefore, agree remarkably with mine, both as to the magnitude of the variation and the time of its maximum effect.

On the other hand, Dr. Nyrén, from a thorough discussion of three series of Pulkowa observations made by Peters, by Gyldén, and by himself, has arrived at discordant results, and concludes that there is no constancy of magnitude or phase in the variation of apparent latitude. Dr. Nyrén's investigation is contained in his paper entitled *Die Polhöhe von Pulkowa*. There appears, therefore, to be this difficulty about the question—that if a short series of observations is used we cannot be sure that we have satisfactorily got rid of accidental errors, or of errors (possibly) depending on the refraction having periods of twelve months, or of six months; and if we discuss a sufficiently long series of observations to eliminate these errors, we make what may be an illegitimate assumption—viz., that the variation remains constant.

The co-latitude of Greenwich deduced from the ten-years' observation of *Polaris* is

$$38^{\circ}\ 31'\ 21''{\cdot}811 \pm 0''{\cdot}011.$$

*Blackheath,*
1880, *May* 10.

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*On a Photograph of Jupiter's Spectrum showing Evidence of Intrinsic Light from that Planet.* By Henry Draper, M.D.

There has been for some years a discussion as to whether the planet *Jupiter* shone to any perceptible extent by his own intrinsic light, or whether the illumination was altogether derived from the Sun. Some facts seem to point to the conclusion that it is not improbable that *Jupiter* is still hot enough to give out light, though perhaps only in a periodic or eruptive manner.

It is obvious that spectroscopic investigation may be usefully employed in the examination of this question and I have incidentally, in the progress of an allied inquiry,* made a photograph which has sufficient interest to be submitted to the inspection of the Astronomical Society.

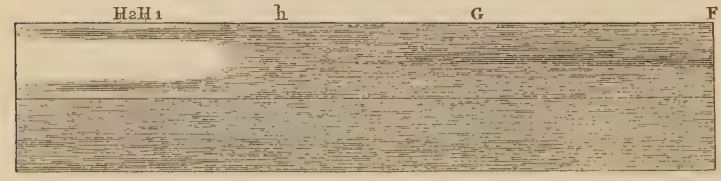
If the light of *Jupiter* is in large part the result of his own incandescence it is certain that the spectrum must differ from that of the Sun, unless the improbable hypothesis is advanced that the same elements, in the same proportions and under the same physical conditions, are present in both bodies. Most of

* See paper *On Photographing the Spectra of the Stars and Planets*, read before the American National Academy of Sciences, Oct. 28, 1879, and published in *Nature*, Nov. 27, 1879, and in the *American Journal of Science*, Dec. 1879.

the photographs I have made of the spectrum of *Jupiter* answer this question decidedly, and from their close resemblance to the spectrum of the Sun indicate that under the average circumstances of observation almost all the light coming to the Earth from *Jupiter* must be merely reflected light originating in the Sun. For this reason I have used the spectrum of *Jupiter* as a reference spectrum on many of my stellar spectrum photographs.

But on one occasion, viz. on September 27, 1879, a spectrum of *Jupiter* with a comparison spectrum of the Moon was obtained which shows a different state of things. Fortunately, owing to the assiduous assistance of my wife, I have a good record of the circumstances under which this photograph was taken, and this will make it possible to connect the aspect of *Jupiter* at the time with the spectrum photograph, though I did not examine *Jupiter* with any care through the telescope that night, and indeed did not have my attention attracted to this photograph till some time afterwards.

I send herewith to the Astronomical Society for examination the original negative, which is just as it was produced, except that it has been cemented with Canada balsam to another piece of glass for protection. Attached to the photograph is an explanatory diagram (*see* figure) intended to point out the peculiarities which are of interest. It will be noticed at once that the main dif-




ference is not due to a change in the number or arrangement of the Fraunhofer lines, but rather to a variation in the strength of the background. In the case of the Moon the background is uniform across the width of the spectrum in any region, but in the case of *Jupiter* the background is fainter in the middle of the width of the spectrum in the region above the line *h*, and stronger in the middle in the region below *h*, especially toward F. The observer must not be confused by the dark portion where the two spectra overlap along the middle of the combined photograph.

In order to interpret this photograph it must be understood that the spectrum of *Jupiter* is produced from an image of the planet thrown upon the slit of the spectroscope by a telescope of 183 inches focal length, the slit being placed approximately in the direction of a line joining the poles of the planet. The spectroscope did not, therefore, integrate the light of the whole disk, but analysed a band at right angles to the equator and

extending across the disk. If either absorption or production of light was taking place on that portion of *Jupiter's* surface, there might be a modification in the intensity of the general background of the photograph spectrum.

A casual inspection will satisfy anyone that such modifications in the intensity of the background are readily perceptible in the original negative. They seem to me to point out two things that are occurring: first, an absorption of solar light in the equatorial regions of the planet; and, second, a production of intrinsic light at the same place. We can reconcile these apparently opposing statements by the hypothesis that the temperature of the incandescent substances producing light at the equatorial regions of *Jupiter* did not suffice for the emission of the more refrangible rays, and that there were present materials which can absorb those rays from the sunlight falling on the planet.

If the spectrum photograph exhibited only the absorption phenomenon above *h* the interest attached to it would not be great, because a physicist will readily admit from theoretical considerations that such might be the case owing to the coloured belts of the planet. But the strengthening of the spectrum between *h* and F in the portions answering to the vicinity of the equatorial regions of *Jupiter* bears so directly on the problem of the physical condition of the planet as to incandescence that its importance cannot be overrated.

The circumstances under which this photograph was taken were as follows :—Longitude of Observatory, $4^h\ 55^m\ 29^s{\cdot}7$ W. of Greenwich. Night not very steady. *Jupiter* and the Moon differed but little in altitude. *Jupiter's* spectrum was exposed to the photographic plate for 50 minutes, the Moon was exposed for 10 minutes. *Jupiter* was near the meridian. The photograph of *Jupiter's* spectrum was taken between $9^h\ 55^m$ and $10^h\ 45^m$ New York mean time, September 27, 1879.

I have suspected that perhaps there may have been an influence produced by the great coloured patch on Jupiter which has made itself felt in this photograph. It may be that eruptions of heated gases and vapours of various composition, colour, and intensity of incandescence are taking place on the great planet; and a spot which would not be especially conspicuous from its tint to the eye might readily modify the spectrum in the manner spoken of above.

*Central Solar Eclipses in Great Britain during* 1,000 *years.*
By the Rev. S. J. Johnson.

The following list shows all the central eclipses I have been able to find in Great Britain from the thirteenth to the twenty-third century (inclusive), computed by the same approximate method used for the eclipses in the Saxon Chronicle (6th to 12th century). See *Monthly Notices*, April 1873.

| | d | h | | | d | h | |
|---|---|---|---|---|---|---|---|
| 1263 Aug. | 5 | 1¼ | annular | 1737 March | 1 | | annular |
| 1279 April | 12 | 6¼ | annular | 1748 July | 25 | | annular |
| 1310 Jan. | 31 | 1¼ | annular | 1764 April | 1 | | annular |
| 1330 July | 16 | 3¾ | total | 1836 May | 15 | | annular |
| 1411 Aug. | 18 | 6 | annular | 1847 Oct. | 9 | | annular |
| 1433 June | 17 | 3 | total | 1858 March | 15 | | annular |
| 1502 Sept. | 30 | 18¾ | annular | 1927 June | 28 | 17¼ | central |
| 1547 Nov. | 12 | 1¼ | annular | 1999 Aug. | 10 | 22¼ | total |
| 1598 Feb. | 25 | | total | 2090 Sept. | 23 | 5½ | total |
| 1601 Dec. | 24 | 1¼ | annular | 2093 July | 23 | 0¼ | annular |
| 1621 May | 20 | 20¼ | annular | 2135 Oct. | 6 | 19¾ | total |
| 1652 April | 8 | | total | 2151 June | 14 | 6½ | total |
| 1715 May | 2 | 21 | total | 2189 Nov. | 7 | 20½ | total |
| 1724 May | 22 | | total | 2200 April | 14 | 5¼ | central |

Continuing the examination to the year A.D. 2500, or 620 years from the present time, no eclipse appears likely to be quite total at Greenwich, the nearest approaches to this being 2151 June 14, and 2381 July 21, especially in the former instance.

Eclipse of 1279 is, by the tables used, annular at London.

1330. This is apparently total in Scotland for a very short time; and in 1339 July 7 at $\frac{1}{2}^{h}$ the annular phase may have touched the North of Scotland.

1411. Probably annular in Ireland and SW. of England.

On May 8, 1491, the central and annular phase would scarcely escape the mainland of Scotland at the extreme North.

1502. Very soon after sunrise.

1547. Very widely annular, and, by the tables used, London was within the track.

1601. Annular right across England. According to these tables, I also obtain an annular phase for Nidiosia (Drontheim) in Norway, where, in the appendix to Tycho Brahé's *Hist. Cel.* it is said to have been so observed. "Sol ita lunare corpus intra sui complexum comprehenderat, ut lux undiquaque ad marginem diffunderetur."

1621. Narrowly annular in England.

On June 10, 1630, the central phase, which must have been

an extremely narrow track, seems only just to have escaped the SW. coast. (Dr. Bainbridge made this eclipse to begin at Oxford at 5·58 and end at 7·48.)

2135. Seems to be total right across England, and deserves a rigorous computation.

2189. This may be total in the SW. of England.

*Abbenhall Rectory, Gloucester,*
1880, *May* 10.

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*On the Variability of B.A.C.* 2472. By J. Tebbutt, Esq.

My suspicions as to the variability of this star were founded on an attempt to observe its occultation by the Moon on April 22, 1874. In the Occultation List of the *Nautical Almanac*, the star is set down as one of the sixth magnitude. The earliest recorded estimates of magnitude that I have yet been enabled to find are those by Lalande in his catalogue of 47390 stars for 1800. Two independent estimates of this astronomer assign 8 and 8½ as its magnitude. In Taylor's Madras Catalogue for 1835, Robinson's Armagh Catalogue for 1840, and the new Greenwich Nine Year Catalogue, it is put down as of the 6th magnitude, while 6·5 is the estimate in the Washington Catalogue of 10658 stars for 1860. At the present epoch, however, it certainly does not exceed the 8th magnitude. As seen in the 3-inch Transit instrument, it will scarcely bear the faintest illumination of the wires, while B.A.C. 2469, which may be viewed in the same field with it, is distinct under strong illumination. I cannot find that B.A.C. 2472 has ever been suspected as a Variable, although it is situated far north of the equator, and not very distant from several well-known Variables in the constellation *Gemini*.

In conclusion, I may also record my strong suspicions as to the variable character of the star numbered 14571 in Lalande's Catalogue, which is, in fact, only a few minutes of a degree distant from B.A.C. 2472.

*Observatory, Windsor, N.S. Wales,*
1880, *January* 12.

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Diagrams of the position of the Great Southern Comet, as seen Feb. 5–12, from the ship 'Superb,' were received from Mr. Barker, and a communication from Mr. Tebbutt, who saw the head of the Comet, for a few seconds, between clouds, on the evening of Feb. 14; also a set of elements computed by Mr. H. T. Vivian, from Mr. Gill's observations of Feb. 11, 13, 15, and agreeing tolerably well with those computed at Lord Lindsay's Observatory.

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*Observations of Comet* 1880 *b* (Schäberle), *made at Dun Echt Observatory with the Filar Micrometer of the* 15*-inch Refractor.*

| Date. 1880. | Dun Echt Mean Time. h m s | $\delta - *$ $\Delta\alpha$ m s | $\Delta\delta$ ′ | $\alpha\delta$ h m s | $\delta\delta$ ° ′ | Observer. |
|---|---|---|---|---|---|---|
| *a* Apr. 22 | 11 57 45·3 | +1 49·47 | − 9 29·0 | 6 15 39·57 | +72 37 42·3 | T. G. L. |
| *b* 22 | 12 55 40·8 | +3 55·53 | 2 19·2 | 6 15 38·46 | 72 36 4·6 | ,, |
| *a* 22 | 12 55 40·8 | +1 48·12 | 11 5·6 | 6 15 38·22 | 72 36 5·7 | ,, |
| 23 | 10 58 22·9 | − 6·81 | 7 16·3 | 6 15 17·54 | 71 58 15·0 | R. C. |
| 25 | 13 51 15·7 | + 11·80 | 3 34·4 | 6 14 47·18 | 70 32 29·2 | T. G. L. |
| May 8 | 11 6 48·8 | + 3·76 | − 1 26·8 | 6 17 9·16 | +62 45 35·8 | R. C. |

*Adopted Mean Places of Comparison Stars for* 1880·0.

| | $\alpha$ h m s | Reduction. s | $\delta$ ° ′ ″ | Reduction. ″ | |
|---|---|---|---|---|---|
| *a* Apr. 22 | 6 13 47·79 | +2·31 | +72 46 56·6 | +14·7 | Arg.-Oeltz. 6740 |
| *b* 22 | 6 11 40·66 | 2·27 | 72 38 9·1 | 14·7 | ,, 6695 |
| 23 | 6 15 22·08 | 2·27 | 72 5 16·8 | 14·5 | $\frac{3\text{ Bonn. Cat.} + 72^\circ\text{ No. }317 + \text{Arg.-Oeltz. }6767}{4}$ |
| 25 | 6 14 33·26 | 2·12 | 70 35 49·8 | 13·8 | Radcliffe Cat. No. 1707 |
| May 8 | 6 17 3·72 | +1·68 | +62 46 52·6 | +10·0 | Arg.-Oeltz. $\frac{6815+6816}{2}$ |

*Dun Echt Observatory,*
1880, *May* 13.

*Elements of Schäberle's Comet.* By J. R. Hind, F.R.S.

The following elements of the Comet discovered by Schäberle, on April 6, are calculated from an observation at Strasburg, by Professor Winnecke, on April 11, and observations by Major Tupman, on April 27 and May 8; the latter being, no doubt, a very accurate position, from the circumstance of the Comet having passed nearly over a star of the eighth magnitude, which was twice observed in Argelander's Zones; the angle of position changed more than 100° during Major Tupman's comparisons:—

Perihelion Passage, 1880, July, 1·61467 G.M.T.

| | | | |
|---|---|---|---|
| $\pi$ | 112° 7′ 42″·4 | } Apparent Equinox April 25 | |
| $\Omega$ | 257 13 42·4 | | |
| $i$ | 56 55 57·3 | For the middle observation (C−O) $\Delta\lambda \cos\beta = -6''\cdot5$ | |
| Log $q$ | 0·2587892 | $\Delta\beta = -4''\cdot0$ | |

Retrograde.

The coordinate constants for equinox of June 1, are

$$x = r[9\cdot7605836] \sin (v + 192^\circ\ 20'\ 33'')$$
$$y = r[9\cdot9647468] \sin (v + 138\ 53\ 27)$$
$$z = r[9\cdot9563320] \sin (v + 60\ 19\ 11)$$

From the above elements it appears probable that the Comet, after being invisible for some weeks about the perihelion passage, will again come under observation early in August, in the north-east before sun-rise, the theoretical intensity of light increasing from that time until the end of October or beginning of November. According to these elements, the Comet will not arrive at its least distance from the Earth (1·6) until about November 3, when it will be situated south of the belt of *Orion* not far from Rigel—therefore, passing the meridian about $14^h\ 30^m$ and very favourably situated for observation. In the first week of December the intensity of light will have diminished to the same value that it has in the first week in June, but the Comet will be projected on darker sky-ground, and it seems possible that it may be within reach of the telescope until quite the end of the year.

## *On the Longitude of the Observatory, Windsor, N.S. Wales.*

By J. Tebbutt, Esq., Director of the Observatory.

I am indebted to Rear-Admiral Rodgers for the Washington corrections to Hansen's places of the Moon corresponding to occultations observed by me in 1876. I have thus been enabled to apply these corrections, in conjunction with those from Greenwich, to the places corresponding to the occultations of Spica and Antares included in my paper sent to the Society in January last, and also to the places corresponding to three other occultation-phases observed here in 1876. I am desirous that the accompanying table should be regarded as supplementary to that in my previous paper. Due regard being paid to the new longitude-corrections from the occultations of Spica and Antares, it will be found that the mean of the twenty-three occultation results is $+6^{s}\cdot11$ and the longitude of my Observatory becomes $10^{h}\ 3^{m}\ 21^{s}\cdot81$, E. This value differs very slightly from that previously forwarded to the Society, and I think it is as satisfactory as any that can be derived from the Moon's motions alone. I shall await with interest its confirmation, or otherwise, by telegraphic connection with Europe.

| Date of Occultation 1876 | Star Occulted | Windsor Mean Time of Observation | Phase of Occultation | Adopted Corrections to Moon's Tabular Place R.A. | Adopted Corrections to Moon's Tabular Place N.P.D. | Resulting Correction to Longitude |
|---|---|---|---|---|---|---|
| June 2 | Spica | 12 34 7.2 | D | −0·62 | −4.8 | +5·08 |
| 2 | Spica | 13 39 5·2 | R | −0·62 | −4·8 | −0·83 |
| July 3 | Antares | 12 14 41·0 | D | −0·63 | −2·1 | +7·74 |
| 3 | Antares | 13 7 32·9 | R | −0·63 | −2·1 | +3·91 |
| Oct. 28 | λ Aquarii | *7 50 21·6 | D | −0·48 | +3·4 | +5·40 |
| 28 | λ Aquarii | 9 12 51·4 | R | −0·48 | +3·4 | −0·61 |
| 28 | 78 Aquarii | 9 30 39·7 | D | −0·48 | +3·4 | +0·74 |

*Errata in my communications in the R.A.S. Monthly Notices.*

Vol. xxxvi. page 43. *For* entirely complete *read* certainly complete.
„ xxxvi. „ 100. *Insert* 1875 *at the head of the Observations.*
„ xxxviii. „ 11. *Omit* local *before* mean times.
„ xxxviii. „ 331. *For* lower *read* hour.
„ xxxix. „ 322. *For* east *read* west, *and for* south *read* north.

*Observatory, Windsor, N. S. Wales,*
1880, *March*, 15.

* Noted rather late.